Learning Journey: Year 13 Further Maths

1. Core Ch 1: Vectors 1
2. Core Ch 2: Matrices
3. Core Ch 3: Series & Induction
4. Core Ch 11: Vectors 2
5. Mechanics Ch 1: Kinematics
6. Mechanics Ch 2: Forces & Motion
7. Mechanics Ch 3: A Model for Friction
8. Mechanics Ch 4: Moments of Forces
9. Mechanics Ch 5: Work, Energy, Power
10. Mechanics Ch 6: Impulse, Momentum
11. Mechanics Ch 7: Centre of Mass
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16. Core Ch 7: Hyperbolic Functions
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Personalised Learning Checklist (PLC): Year 13 Further Maths

Core Chapter 1: Vectors 1	
Form the vector equation of a line in three dimensions where **a** is the position vector of a point A on the line and **d** is the direction vector of the line: $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{d}$	
Form the Cartesian equation of a line in three dimensions with direction vector $\boldsymbol{d} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$ passing through point A with position vector $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$: $\lambda = \frac{x-a_1}{d_1} = \frac{y-a_2}{d_2} = \frac{z-a_3}{d_3}$	
Know the different ways in which two lines can intersect or not in three dimensional space	
Find out whether two lines in three dimensions are parallel, skew or intersect	
Find the point of intersection of straight lines, if there is one	
Find the angle between two lines, by finding the angle between their direction vectors	
Find the point of intersection of a line and plane by substituting for x, y and z in the equation of the plane to find λ, and then substituting λ back in	
Find the angle between a line and plane by finding the angle between the direction vector of the line and the normal to the plane, then subtracting from 90	

Formula Booklet Extract

Vectors and 3-D geometry

Cartesian equation of a plane is
$n_1 x + n_2 y + n_3 z + d = 0$

Cartesian equation of a line in 3-D is

$$\frac{x-a_1}{d_1} = \frac{y-a_2}{d_2} = \frac{z-a_3}{d_3}$$

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Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: VECTORS AND 3-D SPACE (b)					
Lines	v9	Be able to form and use the equation of a line in 3-D.	In vector and cartesian form. Direction vector.	Line: $\mathbf{r} = \mathbf{a} + t\mathbf{d}$ $\frac{x-a_1}{d_1} = \frac{y-a_2}{d_2}$ $= \frac{z-a_3}{d_3} (=t)$	
	v10	Be able to calculate the angle between two lines.	The angle between two non-perpendicular lines (which may be skew) is the acute angle between their direction vectors.		
	v11	Know the different ways in which two lines can intersect or not in 3-D space.	Two lines intersect at a point or are parallel or skew.		
	v12	Be able to determine whether two lines in three dimensions are parallel, skew or intersect, and to find the point of intersection if there is one.			
	v13	Be able to find the distance between two parallel lines and the shortest distance between two skew lines.	Formula for skew lines will be given, but questions may expect understanding of the underlying principles.		Proof of formula.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: VECTORS AND 3-D SPACE (b)					
Points, lines and planes	Pv14	Be able to find the intersection of a line and a plane.			
	v15	Be able to calculate the angle between a line and a plane.	If they are not perpendicular, the angle between a line and a plane is the acute angle between the line and its orthogonal projection onto the plane.		The language 'orthogonal projection' is not expected.

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Personalised Learning Checklist (PLC): Year 13 Further Maths

Core Chapter 2: Matrices	
Find the determinant of a 2×2 matrix $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ using $\det M = ad - bc$ and explain the geometrical significance	
Find the inverse of a non-singular 2×2 matrix $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$	
Know the terms minor, cofactor, adjoint/adjugate in relation to M = $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ The minor of a_1 is $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} = b_2c_3 - c_2b_3$ The cofactor A_1 = $+\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$ is a minor with its associated place sign, with signs given by $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$ The adjoint or adjugate is a matrix of cofactors Adj M = $\begin{pmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{pmatrix}$	
Find the determinant of a 3×3 matrix and explain the geometrical significance Det M = $a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$ - $a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}$ + $a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$	
Find the inverse of a non-singular 3×3 matrix $M^{-1} = \frac{1}{\det M}\begin{pmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{pmatrix}$	
Know that a matrix is singular if the determinant is zero, and all points are mapped to straight line (2D) or plane (3D)	
Know that a matrix is non-singular is the determinant is non-zero, and then $AA^{-1} = A^{-1}A = I$	
Use matrices to solve simultaneous linear equations	
Use matrices to determine how three planes intersect in three dimensions	

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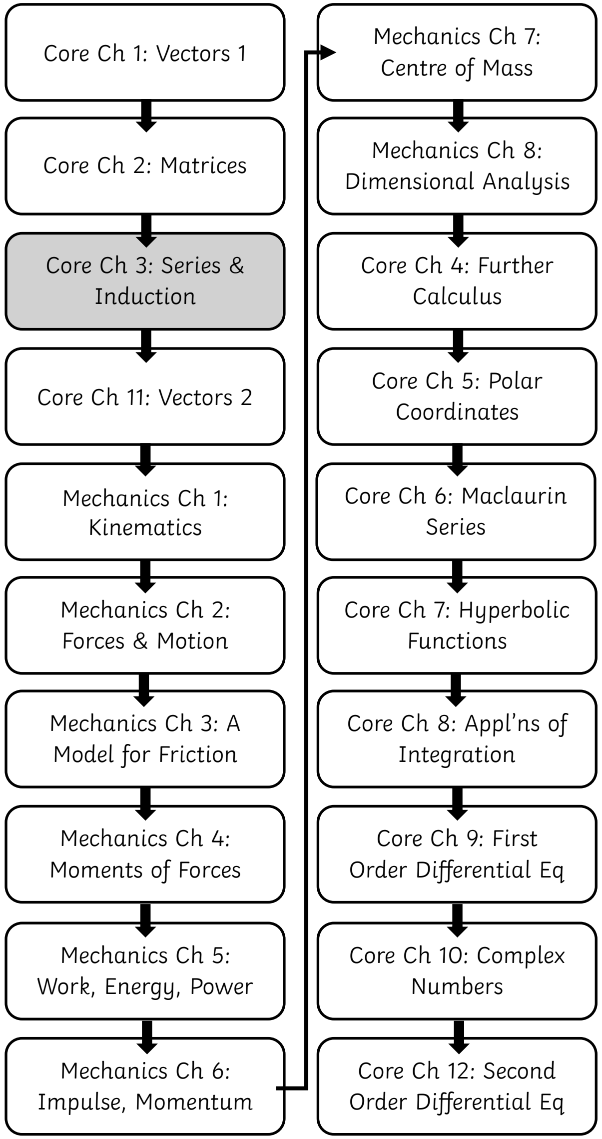
Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: MATRICES AND TRANSFORMATIONS (a)					
Determinant of a matrix	m7	Be able to calculate the determinant of a 2×2 matrix and a 3×3 matrix. Know the meaning of the terms singular and non-singular as applied to matrices.	With a calculator for 3×3 matrices. A singular square matrix is non-invertible and therefore has determinant zero.	$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ or det $\mathbf{M}$ or $\lvert\mathbf{M}\rvert$.	
	m8	Know that the magnitude of the determinant of a 2×2 matrix gives the area scale factor of the associated transformation, and understand the significance of a zero determinant. Interpret the sign of a determinant in terms of orientation of the image.	E.g. Quadrilateral ABCD is labelled clockwise and transformed in 2-D; a negative determinant for the transformation matrix means that the labelling on the image A′B′C′D′ is anticlockwise.		Proof.
	m9	Know that the magnitude of the determinant of a 3×3 matrix gives the volume scale factor of the associated transformation, and understand the significance of a zero determinant. Interpret the sign of a determinant in terms of orientation of the image.	The sign of the determinant determines whether the associated transformation preserves or reverses orientation ('handedness'). E.g. If a triangle ABC is labelled clockwise when seen from point S, then for a negative determinant, the triangle A′B′C′ is anti-clockwise when seen from S′.		Proof
	m10	Know that det(**MN**)=det **M** × det **N** and the corresponding result for scale factors of transformations.	Scale factors in 2-D only.		Algebraic proof.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
Inverses of square matrices	m11	Understand what is meant by an inverse matrix.	Square matrices of any order.	$\mathbf{M}^{-1}$	
	m12	Be able to calculate the inverse of a non-singular 2×2 matrix or 3×3 matrix.	With a calculator for 3×3 matrices. $\det(\mathbf{A}^{-1}) = \dfrac{1}{\det \mathbf{A}}$		
	m13	Be able to use the inverse of a non-singular 2×2 or 3×3 matrix. Relate the inverse matrix to the corresponding inverse transformation.	E.g. to solve a matrix equation and interpret in terms of transformations: find the pre-image of a transformation.		
	m14	Understand and use the product rule for inverse matrices.	$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$		
CORE PURE: MATRICES AND TRANSFORMATIONS (b)					
3×3 matrices	m15	Be able to find the determinant and inverse of a 3×3 matrix without a calculator.	May include algebraic terms.		

Learning Journey: Year 13 Further Maths



Personalised Learning Checklist (PLC): Year 13 Further Maths

Core Chapter 3: Series and Induction	
Sum a simple series using standard formulae for $\sum r$, $\sum r^2$, $\sum r^3$	
Sum a simple series using the method of differences	
Sum a simple series using partial fractions	
Construct and present a proof using mathematical induction for a formula for the nth term of a simple sequence	
Construct and present a proof using mathematical induction for the sum of a simple series	
Construct and present a proof using mathematical induction for the nth power of a matrix	
Construct and present a proof using mathematical induction for a divisibility result	

Formula Booklet Extract

Series

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1) \qquad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Formulas to Learn

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

Specification: OCR Further Mathematics B (MEI) H645

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: PROOF (a)					
Proof	*	Be able to prove mathematical results by deduction and exhaustion, and disprove false conjectures by counter example.	Includes proofs of results used in this specification, where appropriate.		
Induction	Pp4	Be able to construct and present a proof using mathematical induction for given results for a formula for the *n*th term of a sequence, the sum of a series or the *n*th power of a matrix.	The result to be proved will be given. E.g. for the sequence given by $u_1 = 0$, $u_{n+1} = u_n + 2n$ prove that $u_n = n^2 - n$.	u_n, $\sum_{r=1}^{n} r^2$	
CORE PURE: PROOF (b)					
Proof	*	Be able to prove mathematical results by contradiction.			
Induction	Pp5	Be able to construct and present a proof using mathematical induction.	E.g. proofs of divisibility, proof of de Moivre's theorem. The result to be proved will always be given explicitly.		

CORE PURE: SERIES (a)					
Summation of series	Ps1	Be able to use standard formulae for Σr, Σr^2 and Σr^3 and the method of differences to sum series.	Formulae for Σr^2 and Σr^3 will be given but proof could be required, e.g. by induction.	$\sum_{r=1}^{n} r^2$	
CORE PURE: SERIES (b)					
Sequences and series	*	Know the difference between a sequence and a series.			
	*	Know the meaning of the word *converge* when applied to either a sequence or a series.			
Summation of series	Ps2	Be able to sum a simple series using partial fractions.			

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Personalised Learning Checklist (PLC): Year 13 Further Maths

Core Chapter 11: Vectors 2	
Use the vector product in component form to find a vector perpendicular to two given vectors	
Know that $a \times b = \|a\|\|b\| \sin\theta \ \hat{n}$, where a, b and $\hat{n}$, in that order, first a right-handed triple	
Use $\sin\theta = \frac{\|a \times b\|}{\|a\|\|b\|}$ to find the angle between vectors a and b	
Find the shortest distance from a point P to a line with direction vector $\boldsymbol{d}$ and passing through the point A, using $\frac{\|\overrightarrow{AP} \times \boldsymbol{d}\|}{\|\boldsymbol{d}\|}$	
Find the shortest distance from a point $P(x_1, y_1)$ to a line $ax + by + c = 0$ using $\frac{\|ax_1 + by_1 + c\|}{\sqrt{a^2 + b^2}}$	
Find the shortest distance from a point $P(x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$ using $\frac{\|ax_1 + by_1 + cz_1 + d\|}{\sqrt{a^2 + b^2 + c^2}}$	
Find the distance between two parallel lines by picking a point on one lone and finding the shortest distance to the other line	
Find the shortest distance between two skew lines with position vectors a_1 and a_2 and direction vectors d_1 and d_2, using $\left\| \frac{d_1 \times d_2}{\|d_1 \times d_2\|} . (a_1 - a_2) \right\|$	

Formula Booklet Extract

Vector product $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$

$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = |\mathbf{a}||\mathbf{b}| \sin\theta \ \hat{\mathbf{n}}$ where $\mathbf{a}$, $\mathbf{b}$, $\hat{\mathbf{n}}$, in that order, form a right-handed triple.

Distance between skew lines is $\left| \frac{\mathbf{d}_1 \times \mathbf{d}_2}{|\mathbf{d}_1 \times \mathbf{d}_2|} . (\mathbf{a}_1 - \mathbf{a}_2) \right|$ where $\mathbf{a}_1$ is the position vector of a point on the first line and $\mathbf{d}_1$ is parallel to the first line, similarly for the second line.

Distance between point (x_1, y_1) and line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Distance between point (x_1, y_1, z_1) and plane $n_1 x + n_2 y + n_3 z + d = 0$ is $\frac{|n_1 x_1 + n_2 y_1 + n_3 z_1 + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

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Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: VECTORS AND 3-D SPACE (b)					
Vector product	Pv7	Be able to use the vector product in component form to give a vector perpendicular to two given vectors.	Vectors with numerical components only. When a vector perpendicular to two others is required learners should indicate that they are using the vector product but no further working need be shown. Formula will be given; a calculator may be used. $\mathbf{a}\times\mathbf{b}$ $\mathbf{a}\times\mathbf{b}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}=\begin{vmatrix}\mathbf{i} & a_1 & b_1\\\mathbf{j} & a_2 & b_2\\\mathbf{k} & a_3 & b_3\end{vmatrix}$		
Vector products	v8	Be able to use the alternative form for the vector product. Know the significance of $\mathbf{a}\times\mathbf{b}=\mathbf{0}$.	$\mathbf{a}\times\mathbf{b}=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\sin\theta\hat{\mathbf{n}}$ where $\mathbf{a}, \mathbf{b}, \hat{\mathbf{n}}$, in that order, form a right-handed triple. Formula will be given.	The vectors **i, j, k**, in that order, form a right-handed triple.	
	v13	Be able to find the distance between two parallel lines and the shortest distance between two skew lines.	Formula for skew lines will be given, but questions may expect understanding of the underlying principles.		Proof of formula.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: VECTORS AND 3-D SPACE (b)					
Points, lines and planes	Pv14	Be able to find the intersection of a line and a plane.			
	v15	Be able to calculate the angle between a line and a plane.	If they are not perpendicular, the angle between a line and a plane is the acute angle between the line and its orthogonal projection onto the plane.		The language 'orthogonal projection' is not expected.
	v16	Be able to find the distance from a point to a line in 2 or 3 dimensions.	The distance between a point and a line means the shortest distance between them. Formula will be given in 2-D case, but questions may expect understanding of the underlying principles.		Proof of formula.
	v17	Be able to find the distance from a point to a plane.	The distance between a point and a plane means the shortest distance between them. Formula will be given, but questions may expect understanding of the underlying principles.		Proof of formula.

Learning Journey: Year 13 Further Maths

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Personalised Learning Checklist (PLC): Year 13 Further Maths

Mechanics Chapter 1: Kinematics	
Understand and use the terms: distance, displacement, position, speed, velocity, acceleration and their SI units	
Know that displacement, position, velocity and acceleration are vector quantities with magnitude and direction	
Know that distance, speed and time are scalar quantities with magnitude only	
Calculate average speed by dividing total distance by total time taken	
Calculate average velocity by dividing displacement by time taken	
Calculate average acceleration by dividing the change in velocity by time taken	
Draw and interpret distance-time and displacement-time graphs • Use the gradient to find speed and velocity	
Draw and interpret speed-time and velocity-time graphs • Use the gradient to find acceleration • Use the area under the graph to find distance and displacement • Use the area below the axis for negative displacement	
Use the constant acceleration equations (suvat) in problem solving	
Use differentiation in kinematics problems with variable acceleration: $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	
Use integration in kinematics problems with variable acceleration: $v = \int a\,dt$ and $s = \int v\,dt$	

Formula Booklet Extract

Motion in a straight line	Motion in two and three dimensions
$v = u + at$	$\mathbf{v} = \mathbf{u} + \mathbf{a}t$
$s = ut + \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
$s = \frac{1}{2}(u + v)t$	$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$
$v^2 = u^2 + 2as$	-
$s = vt - \frac{1}{2}at^2$	$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

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No additional content since the Kinematics work does not go beyond the A Level Maths course, shown below from OCR Mathematics B (MEI) H640

The work on Kinematics in 2D using vectors is covered in Maths in Spring of Year 13, so will not have been covered when it is taught in Further Maths

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS: KINEMATICS IN 1 DIMENSION (1)					
Motion in 1 dimension	Mk1	Understand and use the language of kinematics.	Position, displacement, distance travelled; speed, velocity; acceleration, magnitude of acceleration; relative velocity (in 1-dimension). Average speed = distance travelled ÷ elapsed time Average velocity = overall displacement ÷ elapsed time		
	k2	Know the difference between position, displacement, distance and distance travelled.			
	k3	Know the difference between velocity and speed, and between acceleration and magnitude of acceleration.			
Kinematics graphs	k4	Be able to draw and interpret kinematics graphs for motion in a straight line, knowing the significance (where appropriate) of their gradients and the areas underneath them.	Position-time, displacement-time, distance-time, velocity-time, speed-time, acceleration-time.		
Calculus in kinematics	k5	Be able to differentiate position and velocity with respect to time and know what measures result.		$v = \frac{\mathrm{d}r}{\mathrm{d}t}, a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 r}{\mathrm{d}t^2}$	
	k6	Be able to integrate acceleration and velocity with respect to time and know what measures result.		$r = \int v\,\mathrm{d}t, v = \int a\,\mathrm{d}t$	
Constant acceleration formulae	k7	Be able to recognise when the use of constant acceleration formulae is appropriate.	Learners should be able to derive the formulae.	$s = ut + \frac{1}{2}at^2$ $s = vt - \frac{1}{2}at^2$ $v = u + at$ $s = \frac{1}{2}(u+v)t$ $v^2 - u^2 = 2as$	
Problem solving	k8	Be able to solve kinematics problems using constant acceleration formulae and calculus for motion in a straight line.			

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS: KINEMATICS IN 2 DIMENSIONS (2)					
Motion in 2 dimensions	Mk9	Understand the language of kinematics appropriate to motion in 2 dimensions. Know the difference between, displacement, distance from and distance travelled; velocity and speed, and between acceleration and magnitude of acceleration.	Position vector, relative position. Average speed = distance travelled ÷ elapsed time Average velocity = overall displacement ÷ elapsed time		Relative velocity
	k10	Be able to extend the scope of techniques from motion in 1 dimension to that in 2 dimensions by using vectors.	The use of calculus and the use of constant acceleration formulae.	$\mathbf{a} = \dot{\mathbf{v}} = \frac{d\mathbf{v}}{dt}, \mathbf{v} = \dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt}$ $\mathbf{r} = \int \mathbf{v}dt, \mathbf{v} = \int \mathbf{a}dt$ $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$	Vector form of $v^2 - u^2 = 2as$
	k11	Be able to find the cartesian equation of the path of a particle when the components of its position vector are given in terms of time.			
	k12	Be able to use vectors to solve problems in kinematics.	Includes relative position of one particle from another. Includes knowing that the velocity vector gives the direction of motion and the acceleration vector gives the direction of resultant force.		

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Mechanics Chapter 2: Forces and Motion	
Draw a diagram showing the forces acting on a body	
Apply Newton's laws of motion to problems in one or more dimensions • Every object continues in a state of rest or uniform motion unless acted on by an external force • Resultant force = mass x acceleration or $\boldsymbol{F} = m\boldsymbol{a}$ • When one object exerts a force on another there is always a reaction force equal and in the opposite direction to the acting force	
Resolve a force into components having selected suitable directions for resolution $\boldsymbol{F} = F\cos\theta\,\boldsymbol{i} + F\sin\theta\,\boldsymbol{j}$	
Find the resultant of several concurrent forces	
Realise that a particle is in equilibrium under a set of concurrent forces if and only if the resultant force is zero	
Know that a closed polygon may be drawn to represent the forces acting on a particle in equilibrium	
Formulate equations for equilibrium by resolving forces in suitable directions	
Formulate the equation of motion of a particle which is being acted on by several forces	
Know that the contact between two surfaces is lost when the normal reaction force becomes zero	

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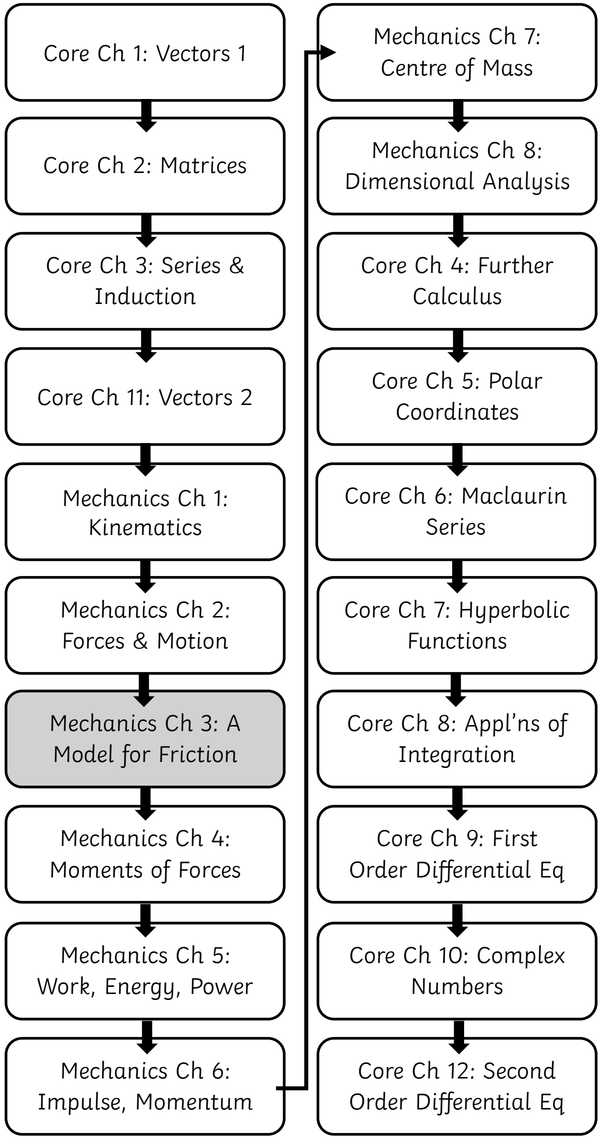
Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: FORCES (a)					
The language of forces	*	Understand the language relating to forces. Understand that the value of the normal reaction depends on the other forces acting and why it cannot be negative.	Weight, tension, thrust (or compression), normal reaction (or normal contact force), frictional force, resistance. Driving force, braking force[1]. NB weight is not considered to be a resistive force.		
	[1]The driving force of a car, bicycle, train engine etc is modelled as a single external force. Similarly for a braking force. These are actually frictional forces acting at the point(s) of contact with the road or track. The internal processes which cause these forces are not considered.				
Friction	Md1	† Understand that bodies in contact may be subject to a frictional force as well as a normal contact force (normal reaction), and be able to represent the situation in an appropriate force diagram.	Smooth is used to mean frictionless.		
Vector treatment of forces	d6	† Be able to resolve a force into components and be able to select suitable directions for resolution.	E.g. horizontally and vertically, or parallel and perpendicular to an inclined plane.		
	d7	† Be able to find the resultant of several concurrent forces by vector addition.	Graphically or by adding components.		

Specification	Ref.	Learning Outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: FORCES (a)					
Equilibrium of a particle	Md8	† Know that a particle is in equilibrium under a set of concurrent forces if and only if their resultant is zero.			
	d9	† Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium.	E.g. a triangle of forces.		
	d10	† Be able to formulate and solve equations for equilibrium by resolving forces in suitable directions, or by drawing and using a polygon of forces.	Questions will not be set that require Lami's theorem but learners may quote and use it where appropriate.		

Learning Journey: Year 13 Further Maths

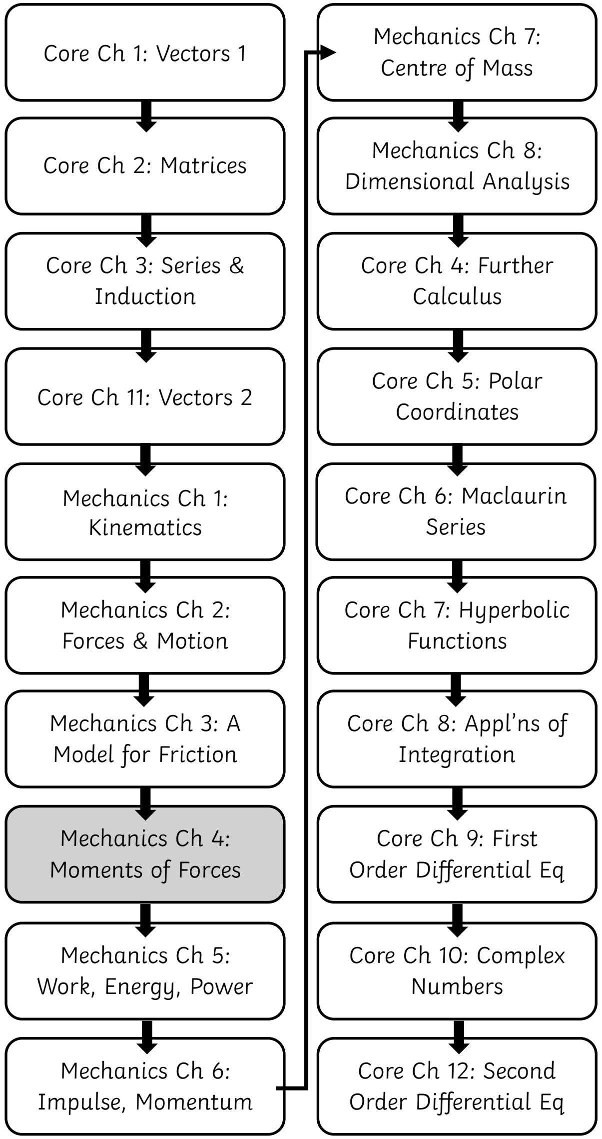


Personalised Learning Checklist (PLC): Year 13 Further Maths

Mechanics Chapter 3: A Model for Friction	
Understand that the total contact force between surfaces may be expressed in terms of frictional force and normal reaction	
Be able to draw a force diagram to represent a situation involving friction	
Know that a frictional force acts in the direction to oppose sliding	
Model the frictional force as $F \leq \mu R$ where μ is the coefficient of friction $F < \mu R$ when there is no sliding except in limiting equilibrium $F = \mu R$ in limiting equilibrium (at the point of sliding) $F = \mu R$ when sliding occurs	
Know how to apply Newton's laws of motion to situations involving friction	
Know the size of the normal reaction, and possibly friction, is affected by any other force with a component perpendicular to the surface on which sliding may occur	
Be able to derive and use the result that a body on a rough slope inclined at angle α to the horizontal is on the point of slipping if $\mu = \tan\alpha$	
Know that the angle for which a body is about to slide down a slope is called the angle of friction, often denoted by λ where $\tan\lambda = \mu$	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: FORCES (a)					
The language of forces	*	Understand the language relating to forces. Understand that the value of the normal reaction depends on the other forces acting and why it cannot be negative.	Weight, tension, thrust (or compression), normal reaction (or normal contact force), frictional force, resistance. Driving force, braking force[1]. NB weight is not considered to be a resistive force.		
	[1]The driving force of a car, bicycle, train engine etc is modelled as a single external force. Similarly for a braking force. These are actually frictional forces acting at the point(s) of contact with the road or track. The internal processes which cause these forces are not considered.				
Friction	Md1	† Understand that bodies in contact may be subject to a frictional force as well as a normal contact force (normal reaction), and be able to represent the situation in an appropriate force diagram.	Smooth is used to mean frictionless.		
	d2	† Understand that the total contact force between surfaces may be expressed in terms of a frictional force and a normal contact force (normal reaction).			
	d3	† Understand that the frictional force may be modelled by $F \leq \mu R$ and that friction acts in the direction to oppose sliding. Model friction using $F = \mu R$ when sliding occurs.	Limiting friction. The definition of μ as the ratio of the frictional force to the normal contact force.	Coefficient of friction is μ.	The term angle of friction.
	d4	Be able to derive and use the result that a body on a rough slope inclined at an angle α to the horizontal is on the point of slipping if $\mu = \tan\alpha$.			
	d5	† Be able to apply Newton's laws to situations involving friction.			

Learning Journey: Year 13 Further Maths

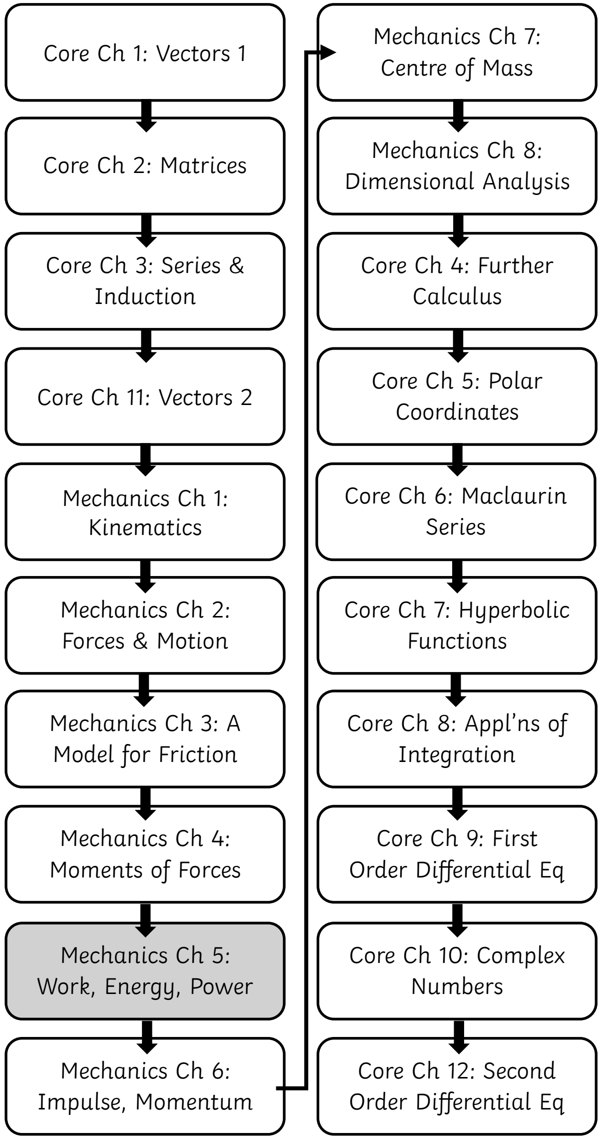


Personalised Learning Checklist (PLC): Year 13 Further Maths

Mechanics Chapter 4: Moments of Forces	
Draw a force diagram for a rigid body	
Calculate the moment of a force about a point or axis using the product Fd where d is the perpendicular distance from the O to the line of action of the force	
Calculate the moment of a force acting by first resolving the force into components and finding the product of that component which does not go through O and its distance from O	
Be able to find the resultant of a set of parallel forces	
Know how different types of lever work • Class 1 where the fulcrum is within the lever (eg scissors or see-saw) • Class 2 where the fulcrum is at one end of the lever (eg wheelbarrow or bottle opener)	
Know the meaning of the word couple – a pair of forces with zero resultant, but non-zero total moment, therefore causing rotation	
Know that an object is in equilibrium if the resultant of all the applied forces acting on it is zero and the sum of their moments about any point is also zero	
Be able to identify how equilibrium can be broken by sliding or toppling	

Specification	Ref.	Learning Outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: FORCES (a)					
Equilibrium of a particle	Md8	† Know that a particle is in equilibrium under a set of concurrent forces if and only if their resultant is zero.			
	d9	† Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium.	E.g. a triangle of forces.		
	d10	† Be able to formulate and solve equations for equilibrium by resolving forces in suitable directions, or by drawing and using a polygon of forces.	Questions will not be set that require Lami's theorem but learners may quote and use it where appropriate.		
Equilibrium of a rigid body	d11	Be able to draw a force diagram for a rigid body.	In cases where the particle model is not appropriate.		
	d12	Understand that a system of forces can have a turning effect on a rigid body.	E.g. a lever.		
	d13	Know the meaning of the term couple.	A couple is not about a particular axis.		
	d14	Be able to calculate the moments about a fixed axis of forces acting on a body. Be able to calculate the moment of a couple.	Both as the product of force and perpendicular distance of the axis from the line of action of the force, and by first resolving the force into components. Take account of a given couple when taking moments.		Vector treatment.
	d15	Understand and be able to apply the conditions for equilibrium of a rigid body.	The resultant of all the applied forces is zero and the sum of their moments about any axis is zero. Three forces in equilibrium must be concurrent or parallel. Situations may involve simple uniform 3-D objects, such as a cuboid, whose centre of mass can be written down by considering symmetry. E.g. infer the existence of a couple by consideration of equilibrium and calculate its size.		
	d16	Be able to identify whether equilibrium will be broken by sliding or toppling.	E.g. a cuboid on an inclined plane.		

Learning Journey: Year 13 Further Maths



Personalised Learning Checklist (PLC): Year 13 Further Maths

Mechanics Chapter 5: Work, Energy and Power	
Understand the language related to work, energy and power	
Know that kinetic energy is the energy a body possesses due to its motion (scalar)	
Calculate kinetic energy using: $KE = \frac{1}{2}mv^2$ where v is the speed	
Know that work has the same units as energy, and is a measure of the change in energy due to a force (the force has done work)	
Calculate the work done by a force which moves either along its line of action or at an angle to it: Work done = constant force x distance moved in that direction	
Know that potential energy is the energy a body possesses due to its position – a stored energy (scalar quantity)	
Calculate gravitational potential energy using $GPE = mgh$ where m is the mass and h is the height above O	
Know that GPE is the work done against the force of gravity in raising a body	
Understand and use the work-energy principle: The total work done by all the forces acting on a body is equal to the increase in kinetic energy of the body	
Understand and use the principle of conservation of energy – mechanical energy (KE + GPE) is conserved when no forces other than gravity do work	
Know that power is the rate of doing work and = Fv	
Know that average power $= \frac{total\ work\ done}{total\ time\ taken}$	
Know that the SI unit for energy is the joule and for power is the watt	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: WORK, ENERGY AND POWER (a)					
The language of work, energy and power	Mw1	Understand the language relating to work, energy and power.	Work, energy, mechanical energy, kinetic energy, potential energy, conservative force, dissipative force, driving force, resistive force. Power of a force, power developed by a vehicle[1].		
	[1]In an examination question 'the power developed by a car' (or a bicycle or train engine) means the useful, or available, power. It is the power of the driving force; it is not the power developed by the engine, some of which is lost in the system.				
Concepts of work and energy	w2	Be able to calculate the work done by a force which moves along its line of action.			The use of calculus for variable forces.
	w3	Be able to calculate the work done by a force which moves at an angle to its line of action.	Zero work is done by a force acting perpendicular to displacement.		Use of scalar product $\mathbf{F}.\mathbf{s}$.
	w4	Be able to calculate kinetic energy.		$\text{KE} = \frac{1}{2}mv^2$	
	w5	Be able to calculate gravitational potential energy.	Relative to a defined zero level.	$\text{GPE} = mgh$	
The work-energy principle	w6	Understand when the principle of conservation of energy may be applied and be able to use it appropriately.	E.g. the maximum height of a projectile, a particle sliding down a smooth curved surface, a child swinging on a rope.		
	w7	Understand and use the work-energy principle.	The total work done by all the external forces acting on a body is equal to the increase in the kinetic energy of the body. E.g. a particle sliding down a rough curved surface.		
Power	w8	Understand and use the concept of the power of a force as the rate at which it does work.	Power = (force) × (component of velocity in the direction of the force). The concept of average power as (work done) ÷ (elapsed time). E.g. finding the maximum speed of a vehicle.		

Learning Journey: Year 13 Further Maths

- Core Ch 1: Vectors 1
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- Core Ch 11: Vectors 2
- Mechanics Ch 1: Kinematics
- Mechanics Ch 2: Forces & Motion
- Mechanics Ch 3: A Model for Friction
- Mechanics Ch 4: Moments of Forces
- Mechanics Ch 5: Work, Energy, Power
- **Mechanics Ch 6: Impulse, Momentum**
- Mechanics Ch 7: Centre of Mass
- Mechanics Ch 8: Dimensional Analysis
- Core Ch 4: Further Calculus
- Core Ch 5: Polar Coordinates
- Core Ch 6: Maclaurin Series
- Core Ch 7: Hyperbolic Functions
- Core Ch 8: Appl'ns of Integration
- Core Ch 9: First Order Differential Eq
- Core Ch 10: Complex Numbers
- Core Ch 12: Second Order Differential Eq

Personalised Learning Checklist (PLC): Year 13 Further Maths

Mechanics Chapter 6: Impulse and Momentum	
Be able to calculate the impulse of a force as a vector and in component form: Impulse $\boldsymbol{J} = \boldsymbol{F}t$ where t is the time for which is acts	
Understand and use the concept of linear momentum and appreciate that it is a vector quantity: Momentum $= m\boldsymbol{v}$ where m = mass and $\boldsymbol{v}$ = velocity	
Understand and use the impulse-momentum equation: Impulse = final momentum – initial momentum or Impulse $= mv - mu$	
Know how to apply the principle of conservation of momentum to direct impacts – where no external force acts, total momentum is constant	
Understand Newton's law of impact and know the meaning of coefficient of restitution, e: $e = \frac{speed\ of\ separation}{speed\ of\ approach}$ or speed of separation $= e \times$ speed of approach	
Know and use the fact that $0 \leq e \leq 1$	
Understand the implications of values of 0 and 1 for the coefficient of restitution	
Know that for perfectly elastic collisions there is no energy loss	
Know that for perfectly inelastic collisions the energy loss is the largest it can be	
Understand that when the coefficient of restitution is less than 1, energy is not conserved during an impact	
Be able to find the loss of kinetic energy during a direct impact	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: MOMENTUM and IMPULSE (a)					
Momentum and impulse treated as vectors	Mi1	Be able to calculate the impulse of a force as a vector and in component form.	Impulse = force × time over which it acts.		The use of calculus for variable forces.
	i2	Understand and use the concept of linear momentum and appreciate that it is a vector quantity.			
	i3	Understand and use the impulse-momentum equation.	The total impulse of all the external forces acting on a body is equal to the change in momentum of the body. Use of relative velocity in one dimension is required.		
Conservation of linear momentum	i4	Understand and use the principle that a system subject to no external force has constant total linear momentum and that this result may be applied in any direction.	The impulse of a finite external force (e.g. friction) acting over a very short period of time (e.g. in a collision) may be regarded as negligible. Application to collisions, coalescence and a body dividing into one or more parts.		
Direct impact	i5	Understand the term direct impact and the assumptions made when modelling direct impact collisions[1].	E.g. a collision between an ice hockey puck and a straight rink barrier: puck moving perpendicular to barrier. E.g. a collision between two spheres moving along their line of centres. E.g. a collision between two railway trucks on a straight track.		Any situation with rotating objects.

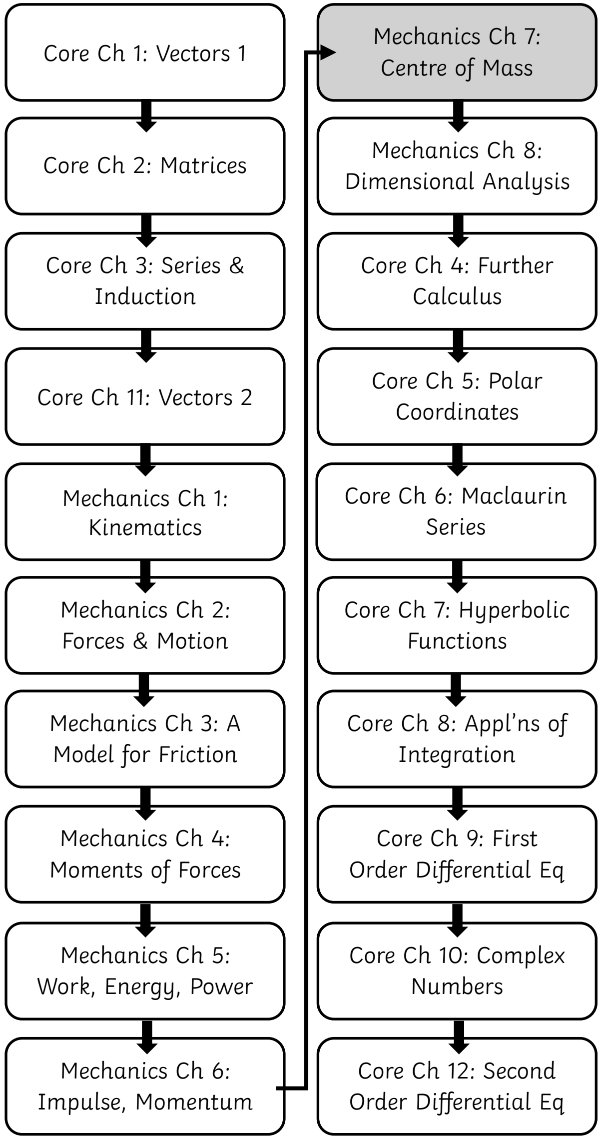
[1]Assumptions when modelling direct impact collisions

This note explains the implicit assumptions made in examination questions when modelling direct impact collisions. Learners may be asked about these assumptions.
An *object* means a real-world object. It may be modelled as a *particle* or a *body*.

- If the non-fixed objects involved in collisions may be modelled as particles, then all the motion and any impulses due to the collisions act in the same straight line.
- If the non-fixed objects involved in collisions may be modelled as bodies then these bodies will be uniform bodies with spherical or circular symmetry.
- The impulse of any collision between such bodies acts on the line joining their centres, and the motion takes place along this line.
  These assumptions ensure that the collision happens at a point and that no angular momentum is created, hence none of the objects starts to rotate.
- The impulse of any collision between such a body, or a particle, and a plane (e.g. a wall or floor) acts in a direction perpendicular to the plane.
  For a direct impact the motion of the object is also in the direction perpendicular to the plane.
- Objects do not rotate before or after the collision. Rotating objects are beyond this specification.

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: MOMENTUM and IMPULSE (a)					
Direct impact (cont)	Mi6	Be able to apply the principle of conservation of linear momentum to direct impacts within a system of bodies.			
	i7	Know the meanings of Newton's Experimental Law and of coefficient of restitution when applied to a direct impact.	Newton's Experimental Law is: the speed of separation is $e \times$ the speed of approach where e is known as the coefficient of restitution.	Coefficient of restitution is e.	
	i8	Understand the significance of $e = 0$.	The bodies coalesce. The collision is inelastic.		
	i9	Be able to apply Newton's Experimental Law in modelling direct impacts.	E.g. between a particle and a wall. E.g. between two discs.		
	i10	Be able to model situations involving direct impact using both conservation of linear momentum and Newton's Experimental Law.			
	i11	Understand the significance of $e = 1$.	The collision is perfectly elastic. Kinetic energy is conserved.		
	i12	Understand that when $e < 1$ kinetic energy is not conserved during impacts and be able to find the loss of kinetic energy.			

Learning Journey: Year 13 Further Maths

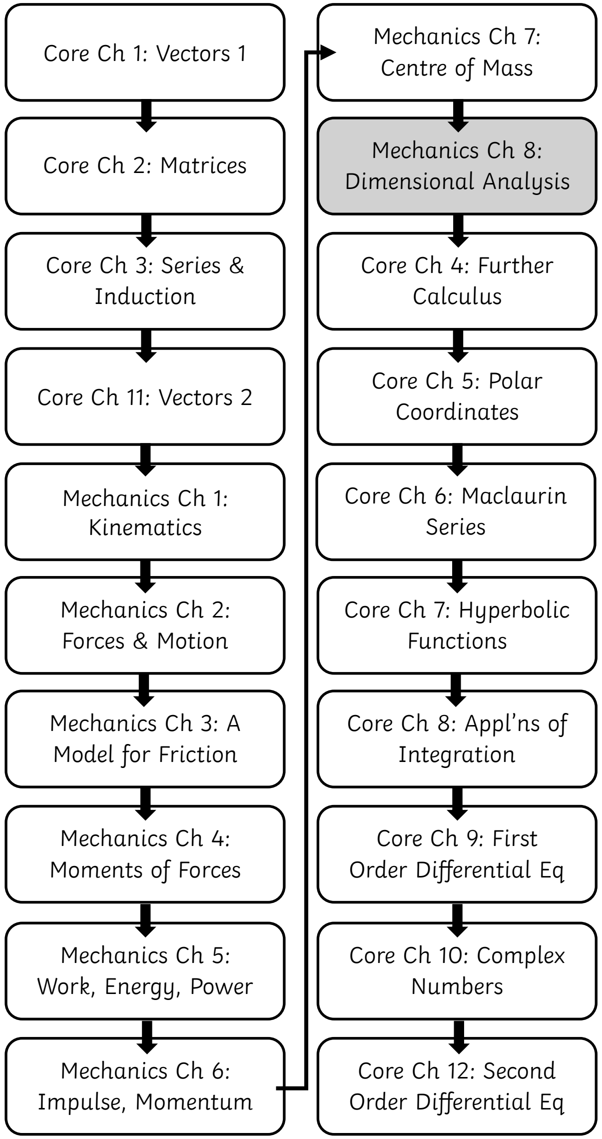


Personalised Learning Checklist (PLC): Year 13 Further Maths

Mechanics Chapter 7: Centres of Mass	
Know that the centre of mass of a body has the property that the moment, about any point, of the whole mass of the body taken at the centre of mass is equal to the sum of moments of the various particles comprising the body $M\bar{\boldsymbol{r}} = \sum m_i \boldsymbol{r_i}$ where $M = \sum m_i$	
Know that in one dimension $M\bar{\boldsymbol{x}} = \sum m_i \boldsymbol{x_i}$	
Know that in two dimensions $M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \sum m_i \begin{pmatrix}x_i\\ y_i\end{pmatrix}$	
Know that in three dimensions $M\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix} = \sum m_i \begin{pmatrix}x_i\\ y_i\\ z_i\end{pmatrix}$	
Find the centre of mass of a system of particles given position and mass	
Know the positions of the centre of mass of a composite body	
Find the centre of mass of a simple shape	
Know how to use symmetry when finding a centre of mass	
Know the positions of the centres of mass of a uniform rod, a rectangular lamina and a triangular lamina	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: CENTRE OF MASS (a)					
Locating a centre of mass	MG1	Be able to find the centre of mass of a system of particles of given position and mass.	In 1, 2 and 3 dimensions.	$(\bar{x}, \bar{y}, \bar{z})$ $\left(\sum_i m_i\right)\bar{x} = \sum_i m_i x_i$	Non-uniform bodies.
	G2	Know how to locate centre of mass by appeal to symmetry.	E.g. uniform circular lamina, sphere, cuboid		
	G3	Know the positions of the centres of mass of a uniform rod, a rectangular lamina and a triangular lamina.			
	G4	Be able to find the centre of mass of a composite body by considering each constituent part as a particle at its centre of mass.	Composite bodies may be formed by the addition or subtraction of parts. Where a composite body includes parts whose centre of mass the learner is not expected to know, or be able to find, the centre of mass will be given.		
Applications of the centre of mass	G5	Be able to use the position of the centre of mass in situations involving the equilibrium of a rigid body.	For the purpose of calculating its moment, the weight of a body can be taken as acting through its centre of mass. E.g. a suspended object E.g. does an object standing on an inclined plane slide or topple?		

Learning Journey: Year 13 Further Maths



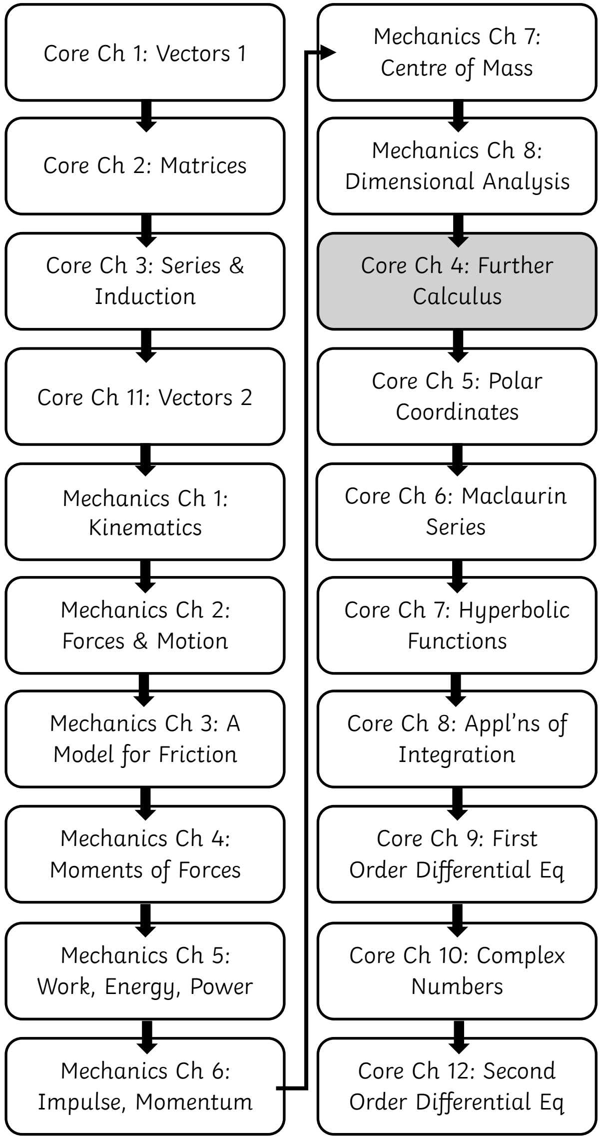
Personalised Learning Checklist (PLC): Year 13 Further Maths

Mechanics Chapter 8: Dimensional Analysis	
Find the dimensions of a quantity in terms of mass M, length L and time T	
Use $[d]$ to denote dimension of quantity d	
Use the dimensions of a quantity to determine its units	
Change the units in which a quantity is given	
Understand that some quantities are dimensionless, such as angles, coefficient of friction, coefficient of restitution	
Use dimensional analysis to check a relationship for consistency	
Use dimensional analysis to determine unknown indices in a proposed formula	
Use a model based on dimensional analysis	

Specification: OCR Further Mathematics B (MEI) H645

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS MINOR: DIMENSIONAL ANALYSIS (a)					
Dimensional consistency	Mq1	Be able to find the dimensions of a quantity in terms of M, L, T.	Know the dimensions of angle and frequency. Work out without further guidance the dimensions of density (mass per unit volume), pressure (force per unit area) and other quantities in this specification. Other kinds of density will be referred to as e.g. mass per unit area. Deduce the dimensions of an unfamiliar quantity from a given relationship.	M, L, T, [ ]	
	q2	Understand that some quantities are dimensionless.			
	q3	Be able to determine the units of a quantity by reference to its dimensions.	And vice versa.		
	q4	Be able to change the units in which a quantity is given.	E.g. density from $\mathrm{kg\,m^{-3}}$ to $\mathrm{g\,cm^{-3}}$.		
	q5	Be able to use dimensional analysis to check the consistency of a relationship.			
Formulating and using models by means of dimensional arguments	q6	Use dimensional analysis to determine unknown indices in a proposed formula.	E.g. for the period of a pendulum.		
	q7	Use a model based on dimensional analysis.	E.g. to find the value of a dimensionless constant. E.g. to investigate the effect of a percentage change in some of the variables.		

Learning Journey: Year 13 Further Maths



Personalised Learning Checklist (PLC): Year 13 Further Maths

Core Chapter 4: Further Calculus	
Evaluate improper integrals where either the integrand is undefined at a value in the interval of integration or the interval of integration extends to infinity	
Differentiate inverse trigonometric functions	
Use the method of partial fractions in integration, including where the denominator has a quadratic factor of the form $ax^2 + c$ and one linear term	
Recognise integrals of functions of the form $(a^2 - x^2)^{-1/2}$ and $(a^2 + x^2)^{-1}$ and be able to integrate related functions by using trigonometric substitutions	

Formula Booklet Extract

Calculus

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$
$\frac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\text{arsinh}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2+a^2})$
$\frac{1}{\sqrt{x^2-a^2}}$	$\text{arcosh}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2-a^2}) \quad (x > a)$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: CALCULUS (b)					
Improper integrals	Pc1	Evaluate improper integrals where either the integrand is undefined at a value in the interval of integration or the interval of integration extends to infinity.	e.g. $\int_{-1}^{1} x^{-\frac{2}{3}}\mathrm{d}x = \int_{-1}^{0} x^{-\frac{2}{3}}\mathrm{d}x + \int_{0}^{1} x^{-\frac{2}{3}}\mathrm{d}x$. e.g. $\int_{1}^{\infty} \mathrm{e}^{-x}\mathrm{d}x$.		
Partial fractions	c4	Be able to use the method of partial fractions in integration, including where the denominator has a quadratic factor of form ax^2+c and one linear term.			
The inverse functions of sine, cosine and tangent	*	Understand the definitions of inverse trigonometric functions.	arcsin: $-\frac{\pi}{2} \le \arcsin x \le \frac{\pi}{2}$. arccos: $0 \le \arccos x \le \pi$. arctan: $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$.	arcsin, $\sin^{-1}$; arccos, $\cos^{-1}$; arctan, $\tan^{-1}$.	
	c5	Be able to differentiate inverse trigonometric functions.			
Use of trigonometric substitutions in integration	c6	Recognise integrals of functions of the form $(a^2-x^2)^{-\frac{1}{2}}$ and $(a^2+x^2)^{-1}$ and be able to integrate related functions by using trigonometric substitutions.	Formulae will be given.		

Learning Journey: Year 13 Further Maths

- Core Ch 1: Vectors 1
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- Mechanics Ch 7: Centre of Mass
- Mechanics Ch 8: Dimensional Analysis
- Core Ch 4: Further Calculus
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- Core Ch 6: Maclaurin Series
- Core Ch 7: Hyperbolic Functions
- Core Ch 8: Appl'ns of Integration
- Core Ch 9: First Order Differential Eq
- Core Ch 10: Complex Numbers
- Core Ch 12: Second Order Differential Eq

Personalised Learning Checklist (PLC): Year 13 Further Maths

Core Chapter 5: Polar Coordinates	
Understand and use polar coordinates	
The principal polar coordinates (r, θ) are those for which $r > 0$ and $-\pi < \theta \leq \pi$	
Convert from polar to Cartesian coordinates and vice versa $x = r\cos\theta$ and $y = r\sin\theta$ $r = \sqrt{x^2+y^2}$ and $\theta = \arctan\frac{y}{x}$ $(\pm\pi)$ if necessary	
Sketch curves with simple polar equations in the form r = f(θ)	
Find the area enclosed by a polar curve-	

Formula Booklet Extract

Area of sector enclosed by polar curve is $\frac{1}{2}\int r^2\,\mathrm{d}\theta$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: POLAR COORDINATES (b)					
Polar coordinates in two dimensions	PP1	Understand and use polar coordinates (r, θ) and be able to convert from polar to cartesian coordinates and vice-versa.	θ in radians.	Pole, initial line.	
	P2	Be able to sketch curves with simple polar equations where r is given as a function of θ.	e.g. $r = a(1 + \cos\theta)$, $r = a\cos 2\theta$.	$r > 0$ continuous line. $r < 0$ broken line.	
	P3	Be able to find the area enclosed by a polar curve.	Using $\frac{1}{2}\int r^2 \mathrm{d}\theta$.		

Learning Journey: Year 13 Further Maths

1. Core Ch 1: Vectors 1
2. Core Ch 2: Matrices
3. Core Ch 3: Series & Induction
4. Core Ch 11: Vectors 2
5. Mechanics Ch 1: Kinematics
6. Mechanics Ch 2: Forces & Motion
7. Mechanics Ch 3: A Model for Friction
8. Mechanics Ch 4: Moments of Forces
9. Mechanics Ch 5: Work, Energy, Power
10. Mechanics Ch 6: Impulse, Momentum
11. Mechanics Ch 7: Centre of Mass
12. Mechanics Ch 8: Dimensional Analysis
13. Core Ch 4: Further Calculus
14. Core Ch 5: Polar Coordinates
15. **Core Ch 6: Maclaurin Series**
16. Core Ch 7: Hyperbolic Functions
17. Core Ch 8: Appl'ns of Integration
18. Core Ch 9: First Order Differential Eq
19. Core Ch 10: Complex Numbers
20. Core Ch 12: Second Order Differential Eq

Personalised Learning Checklist (PLC): Year 13 Further Maths

Chapter 6: Maclaurin Series	
Find the Maclaurin series of a function, including the general term	
Know that a Maclaurin series may converge only for a restricted set of values of x	
Recognise and use the Maclaurin series of standard functions: e^x, ln(1 + x), sin x, cos x and $(1 + x)^n$	

Formula Booklet Extract

Series

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1) \qquad \sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(r)}(0)}{r!}x^r + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \text{ for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots (-1 < x \leq 1)$$

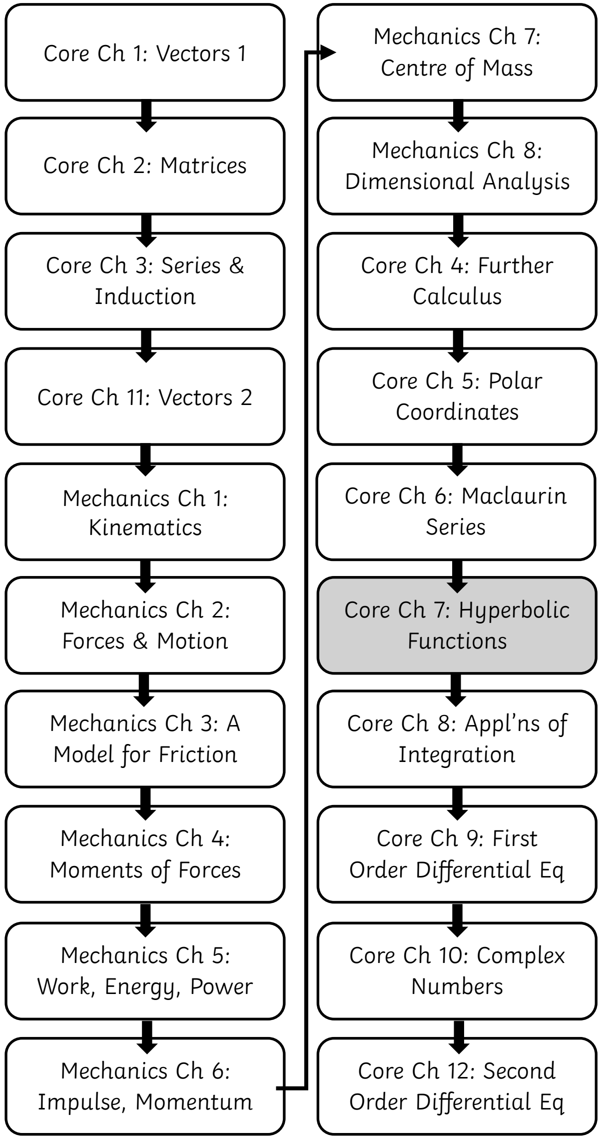
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \text{ for all } x$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

CORE PURE: SERIES (b)					
Sequences and series	*	Know the difference between a sequence and a series.			
	*	Know the meaning of the word *converge* when applied to either a sequence or a series.			
Summation of series	Ps2	Be able to sum a simple series using partial fractions.			
Maclaurin series. Approximate evaluation of a function	s3	Be able to find the Maclaurin series of a function, including the general term.	Use in evaluating approximate values of a function. Error in approximation = approx value – exact value.	Power series.	
	s4	Know that a Maclaurin series may converge only for a restricted set of values of x.			
	s5	Be able to recognise and use the Maclaurin series of standard functions: e^x, $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$.	Identify the set of values of x for which series are valid. Formulae will be given.		Proof of convergence. Complex x.

Learning Journey: Year 13 Further Maths



Personalised Learning Checklist (PLC): Year 13 Further Maths

Chapter 7: Hyperbolic Functions	
Know the definitions of the hyperbolic functions and their domains and ranges, and be able to sketch their graphs	
Understand and use the identity $\cosh^2 x - \sinh^2 x = 1$	
Differentiate and integrate hyperbolic functions	
Understand and use the definitions of the inverse hyperbolic functions and know their domains and ranges	
Derive and use the logarithmic forms of the inverse hyperbolic functions	
Recognise integrals of the form $\frac{1}{\sqrt{x^2-a^2}}$ and $\frac{1}{\sqrt{x^2+a^2}}$ and be able to integrate related functions by using substitutions	

Formula Booklet Extracts

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\operatorname{arsinh} x = \ln[x + \sqrt{(x^2+1)}]$$

$$\operatorname{arcosh} x = \ln[x + \sqrt{(x^2-1)}],\ x \geq 1$$

$$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right),\ -1 < x < 1$$

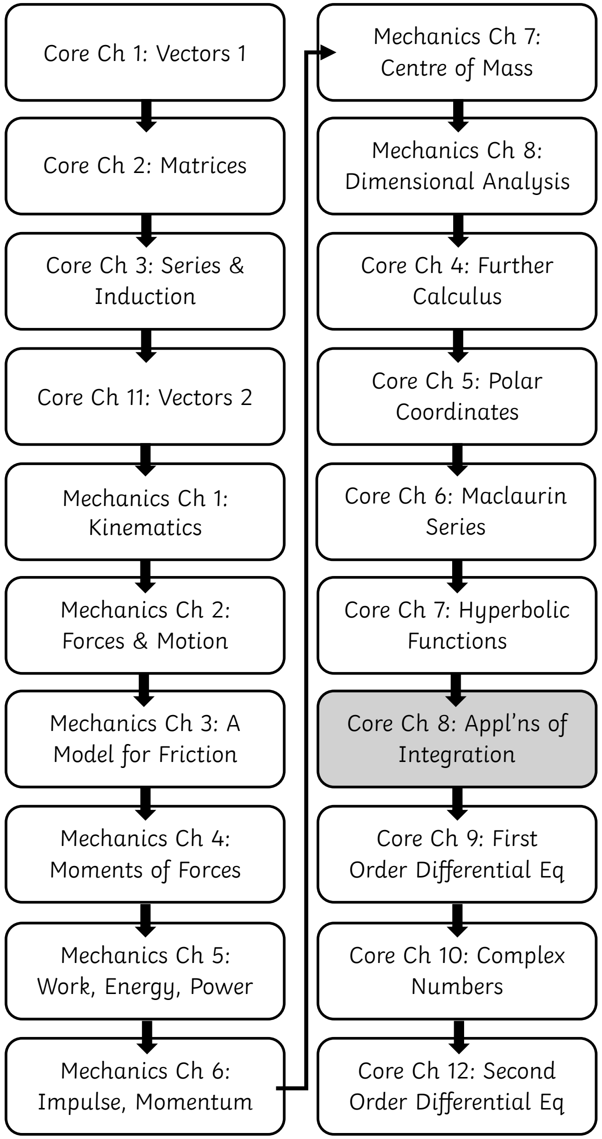
Calculus

$f(x)$	$f'(x)$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

$f(x)$	$\int f(x)\,dx$
$\frac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\operatorname{arsinh}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2+a^2})$
$\frac{1}{\sqrt{x^2-a^2}}$	$\operatorname{arcosh}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2-a^2}) \quad (x > a)$

CORE PURE: HYPERBOLIC FUNCTIONS (b)					
Hyperbolic functions	Pa3	Understand the definitions of hyperbolic functions, know their domains and ranges and be able to sketch their graphs.	$\sinh x = \frac{1}{2}(e^x - e^{-x})$ $\cosh x = \frac{1}{2}(e^x + e^{-x})$ $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$		
	a4	Understand and use the identity $\cosh^2 x - \sinh^2 x = 1$.			Knowledge of other identities.
	a5	Be able to differentiate and integrate hyperbolic functions.			

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: HYPERBOLIC FUNCTIONS (b)					
Inverse hyperbolic functions	Pa6	Understand and be able to use the definitions of the inverse hyperbolic functions and know their domains and ranges.	$\operatorname{arsinh} x$ and $\operatorname{artanh} x$ can take any values but $\operatorname{arcosh} x \geq 0$.		
	a7	Be able to derive and use the logarithmic forms of the inverse hyperbolic functions.	$\operatorname{arsinh} x = \ln[x + \sqrt{(x^2+1)}]$ $\operatorname{arcosh} x = \ln[x + \sqrt{(x^2-1)}], x \geq 1$ $\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), -1 < x < 1$	arsinh, $\sinh^{-1}$; arcosh, $\cosh^{-1}$; artanh, $\tanh^{-1}$.	
	a8	Recognise integrals of functions of the form $(x^2+a^2)^{-\frac{1}{2}}$ and $(x^2-a^2)^{-\frac{1}{2}}$ and be able to integrate related functions by using substitutions.			



Chapter 8: Applications of Integration	
Derive formulae for and calculate volumes of solids generated by rotating a plane region about the x-axis and y- axis	
Evaluate the mean value of a function on a given interval	
Recognise integrals of functions of the form $(a^2 - x^2)^{-1/2}$, $(a^2 + x^2)^{-1}$, $(x^2 - a^2)^{-1/2}$ and $(x^2 + a^2)^{-1/2}$, and be able to integrate related functions by using trigonometric or hyperbolic substitutions	

Formula Booklet Extract

Calculus

$f(x)$	$f'(x)$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$

$f(x)$	$\int f(x)\,dx$
$\frac{1}{\sqrt{a^2-x^2}}$	$\arcsin\left(\frac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{a^2+x^2}}$	$\text{arsinh}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2+a^2})$
$\frac{1}{\sqrt{x^2-a^2}}$	$\text{arcosh}\left(\frac{x}{a}\right)$ or $\ln(x+\sqrt{x^2-a^2}) \quad (x > a)$

The mean value of $f(x)$ on the interval $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: CALCULUS (b)					
Volumes of revolution	c2	Be able to derive formulae for and calculate the volumes of the solids generated by rotating a plane region about the x-axis or the y-axis.	$\pi\int y^2\,\mathrm{d}x$, $\pi\int x^2\,\mathrm{d}y$ and understanding these as the limit of a sum of cylinders.	Volume of revolution.	Axes of rotation other than the x- and y-axes.
Mean value	c3	Understand and evaluate the mean value of a function.	The mean value of $\mathrm{f}(x)$ on the interval $[a, b]$ is $\frac{1}{b-a}\int_a^b \mathrm{f}(x)\,\mathrm{d}x$.		
Use of trigonometric substitutions in integration	c6	Recognise integrals of functions of the form $(a^2-x^2)^{-\frac{1}{2}}$ and $(a^2+x^2)^{-1}$ and be able to integrate related functions by using trigonometric substitutions.	Formulae will be given.		

- Core Ch 1: Vectors 1
- Core Ch 2: Matrices
- Core Ch 3: Series & Induction
- Core Ch 11: Vectors 2
- Mechanics Ch 1: Kinematics
- Mechanics Ch 2: Forces & Motion
- Mechanics Ch 3: A Model for Friction
- Mechanics Ch 4: Moments of Forces
- Mechanics Ch 5: Work, Energy, Power
- Mechanics Ch 6: Impulse, Momentum
- Mechanics Ch 7: Centre of Mass
- Mechanics Ch 8: Dimensional Analysis
- Core Ch 4: Further Calculus
- Core Ch 5: Polar Coordinates
- Core Ch 6: Maclaurin Series
- Core Ch 7: Hyperbolic Functions
- Core Ch 8: Appl'ns of Integration
- **Core Ch 9: First Order Differential Eq**
- Core Ch 10: Complex Numbers
- Core Ch 12: Second Order Differential Eq

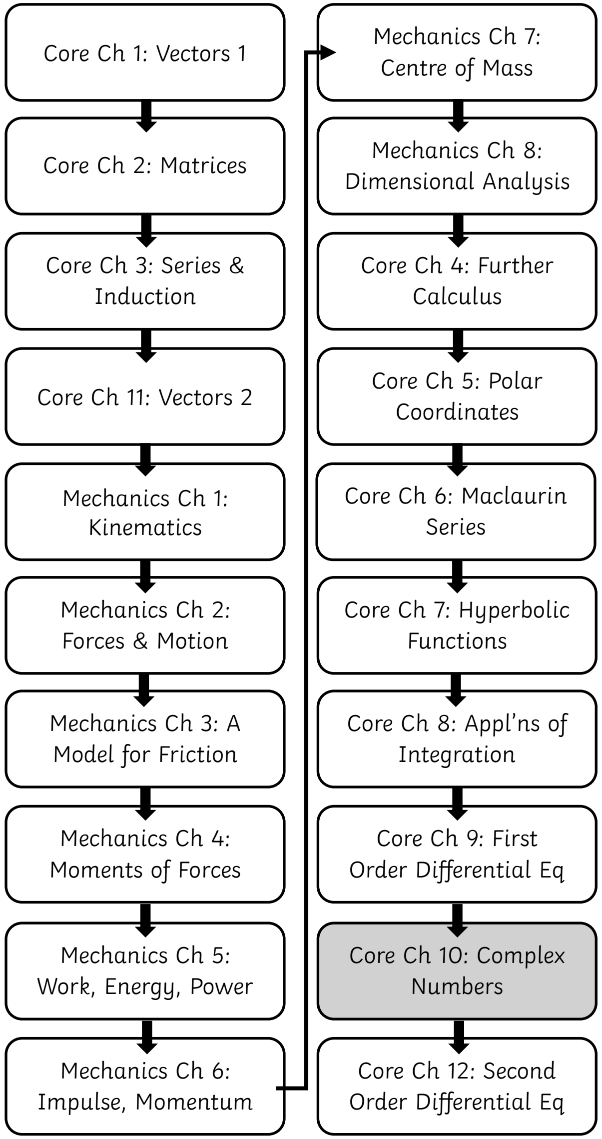
Chapter 9: First Order Differential Equations	
Formulate a differential equation from verbal descriptions involving rates of change	
Use differential equations in modelling	
Know the difference between a general solution and a particular solution	
Find a particular solution to a differential equation by using initial or boundary conditions	
Recognise differential equations which can be solved by separation of variables	
Use separation of variables to solve a first order differential equation, to find both general and particular solutions $\frac{dy}{dx} = f(x)g(y)$ leads to $\int \frac{1}{g(y)} dy = \int f(x) dx$	
Recognise differential equations which can be solved using an integrating factor $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x only	
Find an integrating factor and use it to solve a first order linear differential equation, to find both general and particular solutions To solve $\frac{dy}{dx} + Py = Q$ you multiply throughout by the integrating factor $e^{\int P\,dx}$	

Specification: OCR Further Mathematics B (MEI) H645

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: DIFFERENTIAL EQUATIONS (b)					
Modelling with differential equations	Pp19	Understand how to introduce and define variables to describe a given situation in mathematical terms.			
	p20	Be able to relate 1st and 2nd order derivatives to verbal descriptions and so formulate differential equations.	The differential equations will not be restricted to those which candidates can solve analytically.		
	p21	Know the language of kinematics, and the relationships between the various variables.	Including acceleration $= v\frac{dv}{dx}$.	$v = \frac{dx}{dt} = \dot{x}$ $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}$	
	*	Know Newton's 2nd law of motion.	In the form $F = ma$.		Variable mass.
	p22	Use differential equations in modelling in kinematics and in other contexts.	Sufficient information will be given about contexts which may be unfamiliar.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: DIFFERENTIAL EQUATIONS (b)					
Solutions of differential equations	Pc7	Know the difference between a general solution and a particular solution. Be able to find both general and particular solutions.			
Integrating factor method	c8	Recognise differential equations where the integrating factor method is appropriate.	Equations which can be rearranged into the form $\frac{dy}{dx} + P(x)y = Q(x)$.		
	c9	Be able to find an integrating factor and understand its significance in the solution of an equation.	Integrating factor, $I(x) = e^{\int P(x)dx}$.		
	c10	Be able to solve an equation using an integrating factor and find both general and particular solutions.	E.g. a particular solution through a given point.		

Learning Journey: Year 13 Further Maths



Personalised Learning Checklist (PLC): Year 13 Further Maths

Chapter 10: Complex Numbers	
Find the modulus and argument of a complex number	
Multiply and divide complex numbers in modulus-argument form	
Understand the effect of multiplication by a complex number in an Argand diagram	
Sketch loci of the form $\lvert z - a\rvert = r$, $\arg(z - a) = \theta$ and $\lvert z - a\rvert = \lvert z - b\rvert$ in an Argand diagram	
Use de Moivre's theorem to simplify expressions involving powers of complex numbers	
Find the nth roots of a complex number	
Use de Moivre's theorem to find multiple angle identities	
Use the exponential form of a complex number $z = re^{i\theta}$	
Sum series using de Moivre's theorem	

Formula Booklet Extract

Complex Numbers

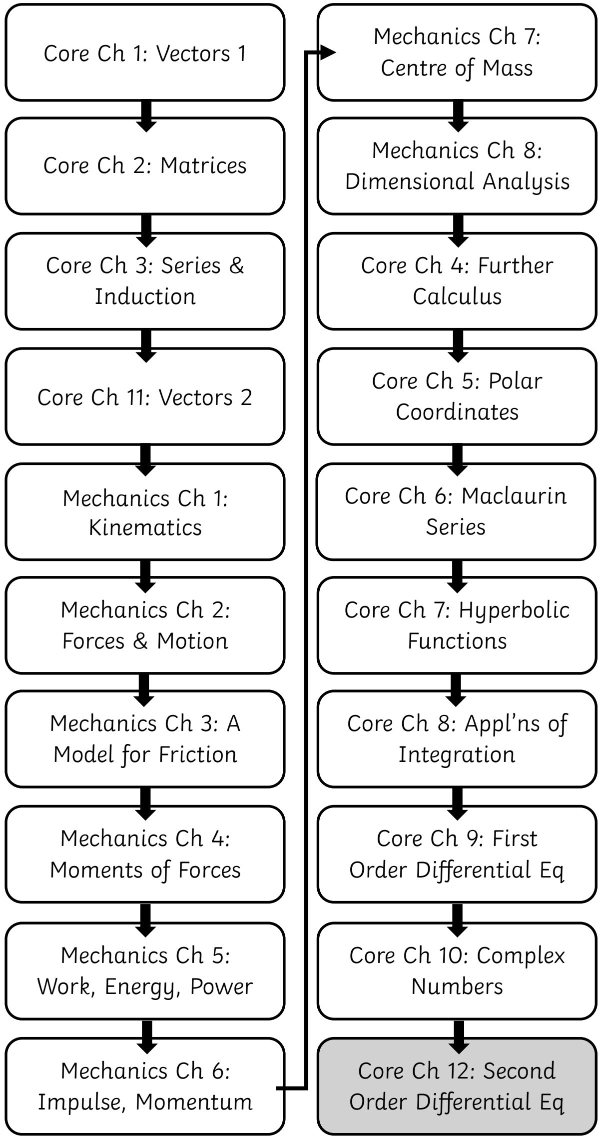
De Moivre's theorem:
$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$

Roots of unity:

The roots of $z^n = 1$ are given by $z = \exp\left(\frac{2\pi k}{n}\mathrm{i}\right)$ for $k = 0, 1, 2, \ldots, n-1$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: COMPLEX NUMBERS (b)					
De Moivre's theorem and simple applications	Pj12	Understand and use de Moivre's theorem.			
	j13	Be able to apply de Moivre's theorem to finding multiple angle formulae and to summing suitable series.	e.g. the expression of $\tan 4\theta$ as a rational function of $\tan\theta$. e.g. finding $\sum_{r=0}^{n} {}_nC_r \cos r\theta$.		
The form $z = re^{i\theta}$	j14	Understand the definition $e^{i\theta} = \cos\theta + i\sin\theta$ and hence the form $z = re^{i\theta}$.			
The *nth* roots of a complex number	j15	Know that every non-zero complex number has n distinct n^{th} roots, and that on an Argand diagram these are the vertices of a regular n-gon.			
	j16	Know that the distinct nth roots of $re^{i\theta}$ are: $r^{\frac{1}{n}}\left[\cos\left(\frac{\theta+2k\pi}{n}\right)+i\sin\left(\frac{\theta+2k\pi}{n}\right)\right]$ for $k = 0, 1 \ldots n-1$.			
	j17	Be able to explain why the sum of all the n^{th} roots is zero.			
Applications of complex numbers in geometry	j18	Understand the effect of multiplication by a complex number on an Argand diagram.	Multiplication by $re^{i\theta}$ corresponds to enlargement with scale factor r with rotation through θ about the origin. e.g. multiplication by i corresponds to a rotation of $\frac{\pi}{2}$ about the origin.		
	j19	Be able to represent complex roots of unity on an Argand diagram.	'Unity' means 1.		
	j20	Be able to apply complex numbers to geometrical problems.	e.g. relating to the geometry of regular polygons.		

Learning Journey: Year 13 Further Maths



Personalised Learning Checklist (PLC): Year 13 Further Maths

Chapter 12: Second Order Differential Equations	
Be able to solve differential equations of the form $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$ using the auxiliary equation method	
Understand and use the relationship between different cases of the solution and the nature of the roots of the auxiliary equation 2 distinct real roots λ_1 and $\lambda_2 \Rightarrow$ complementary function is $y = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$ Repeated root $\alpha \Rightarrow$ complementary function is $y = e^{\alpha x}(A + Bx)$ Complex roots $\lambda = \alpha \pm \beta i \Rightarrow$ complementary function is $y = e^{\alpha x}(A \sin\beta x + B\cos\beta x)$	
Be able to solve the equation for simple harmonic motion, $\frac{d^2y}{dx^2} = -\omega^2 x$ and be able to relate the solution to the motion. The solution is $x = A\sin\omega t + B\cos\omega t$ or $x = a\sin(\omega t + \varepsilon)$ Period of motion is $\frac{2\pi}{\omega}$ and the amplitude is $\sqrt{A^2 + B^2}$	
Be able to model damped oscillations using second order differential equations	
Be able to interpret the solutions of equations modelling damped oscillations in words and graphically $\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} + \omega^2 x = 0$ where $\alpha > 0$ is damped harmonic motion $\alpha = 0$ gives no damping $\alpha^2 - 4\omega^2 > 0$ gives overdamping $\alpha^2 - 4\omega^2 = 0$ gives critical damping $\alpha^2 - 4\omega^2 < 0$ gives underdamping	
Be able to solve differential equations of the form $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = f(x)$ by solving the homogeneous case and adding a particular integral to the complementary function	
Be able to find particular integrals for cases where f(x) is a polynomial, trigonometry or exponential function, including cases where the form of the complementary function affects the form required for the particular integral	
Know that if the trial function for a particular integral is the same as one of the complementary functions, you multiply the trial function by x	
Solved coupled first order simultaneous differential equations involving one independent variable and two dependent variables	

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Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: DIFFERENTIAL EQUATIONS (b)					
Second order differential equations	c11	Be able to solve differential equations of the form $y'' + ay' + by = 0$, using the auxiliary equation.	a and b are constants.	Homogeneous. Complementary function.	
	c12	Understand and use the relationship between different cases of the solution and the nature of the roots of the auxiliary equation.	Discriminant > 0. Discriminant $= 0$. Discriminant < 0.		
	c13	Be able to solve differential equations of the form $y'' + ay' + by = \mathrm{f}(x)$, by solving the homogeneous case and adding a particular integral to the complimentary function.	a and b are constants.		
	c14	Be able to find particular integrals in simple cases. Understand the relationship between different cases of the solution and the nature of the roots of the auxiliary equation.	Cases where $\mathrm{f}(x)$ is a polynomial, trigonometric or exponential function. Includes cases where the form of the complementary function affects the form required for the particular integral.		

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
CORE PURE: DIFFERENTIAL EQUATIONS (b)					
Simple harmonic motion	Pc15	Be able to solve the equation for simple harmonic motion, $\ddot{x} = -\omega^2 x$, and be able to relate the solution to the motion.	Learners may state that they recognise the differential equation is that for SHM, and quote the solution in an appropriate form (e.g. $p\cos(\omega t) + q\sin(\omega t)$ or $A\cos(\omega t - \phi)$), unless specifically required to solve the equation, e.g. by using the techniques of Pc11.	$\lvert A \rvert = \sqrt{p^2 + q^2} =$ amplitude. $T =$ period $= \dfrac{2\pi}{\omega}$	
Damped oscillations	c16	Be able to model damped oscillations using 2nd order differential equations.			
	c17	Be able to interpret the solutions of equations modelling damped oscillations in words and graphically.	The damping will be described as 'over-', 'critical' or 'under-' according to whether the roots of the auxiliary equation are real distinct, equal or complex.	Where applicable, the amplitude refers to the local maximum distance from the equilibrium position. The amplitude decreases with time.	
Simultaneous differential equations	c18	Analyse and interpret model situations with one independent variable and two dependent variables which lead to coupled 1st order simultaneous linear differential equations and find the solution.	Applications include predator-prey models and other population models. E.g. solve by eliminating one variable to produce a single, 2nd order equation.		