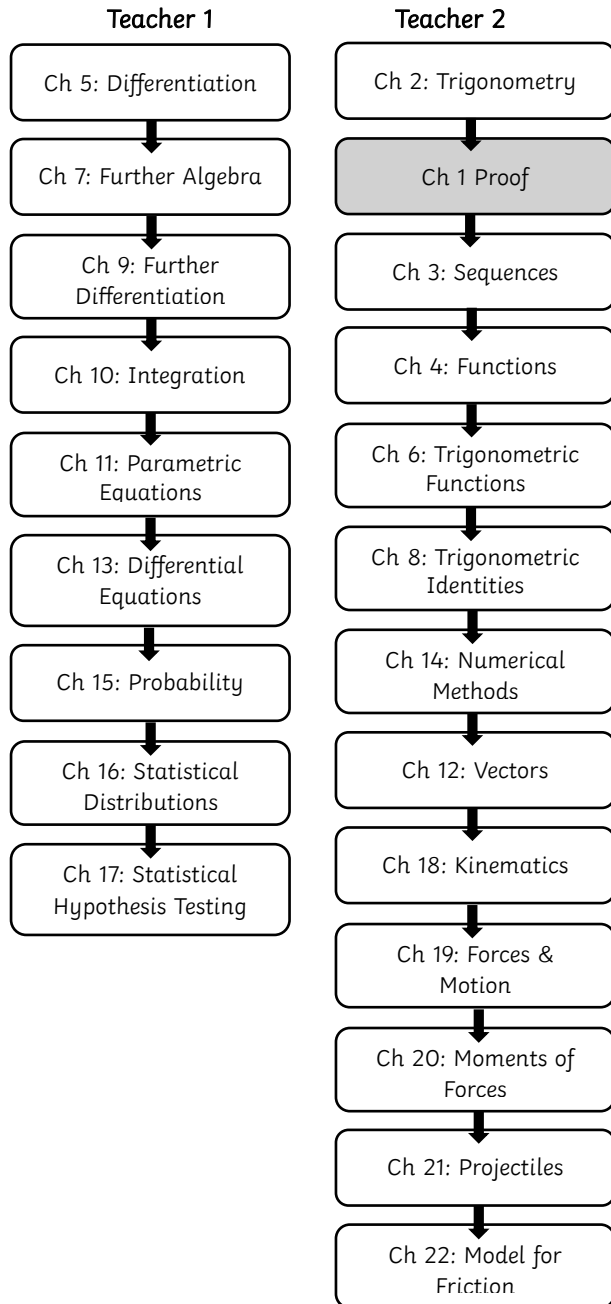


Learning Journey: Year 13 Maths

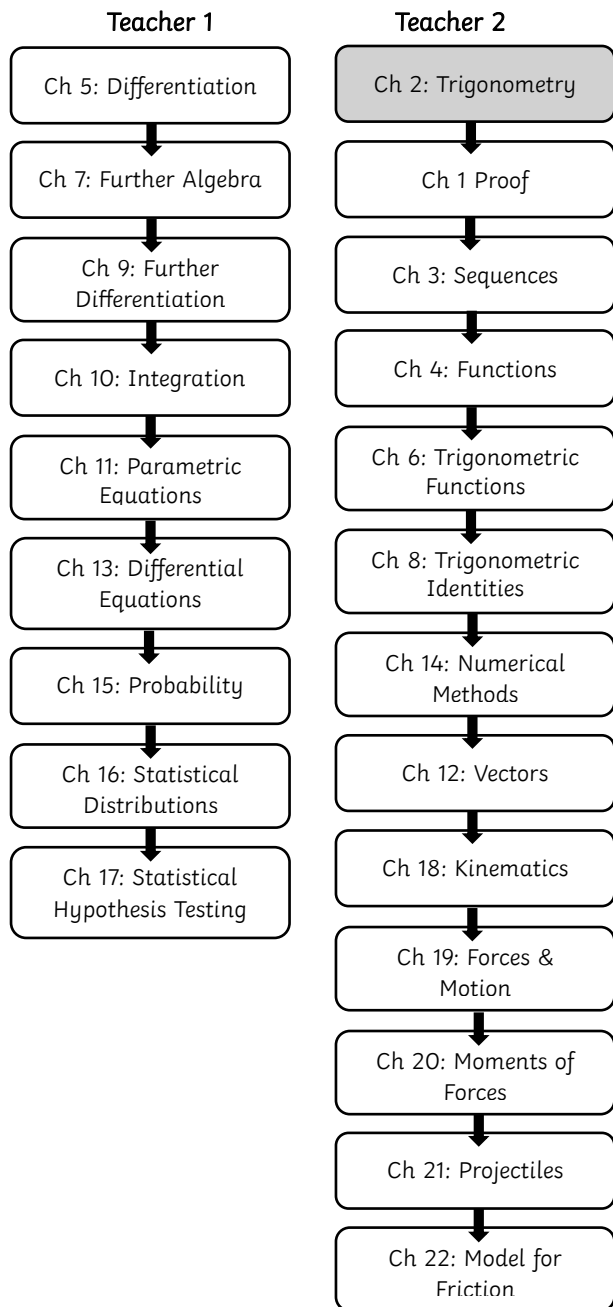


Personalised Learning Checklist (PLC): Year 13 Maths

Chapter 1: Proof	
Understand the problem solving cycle	
Know that simplifying a problem can help to begin in problem solving	
Use algebra to formulate and solve problems	
Interpret your solution in the context of the problem	
Generalise your solution to extend your understanding of a wider problem	
Write your mathematics using appropriate language and symbols <ul style="list-style-type: none"> • Necessary and sufficient • If... and then ... • \Leftrightarrow, \Leftarrow, \Rightarrow and \therefore • Converse of a statement 	
Prove statements by deduction	
Prove statements by exhaustion	
Prove statements by contradiction including: <ul style="list-style-type: none"> • Proof of the irrationality of $\sqrt{2}$ • Proof of the infinity of primes • Application to unfamiliar proofs 	
Disprove statements by finding a counter-example	

Specification: OCR Mathematics B (MEI) H640

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: PROOF (1)					
Proof	Mp1	Understand and be able to use the structure of mathematical proof. Use methods of proof, including proof by deduction and proof by exhaustion.	Proceeding from given assumptions through a series of logical steps to a conclusion.		
	p2	Be able to disprove a conjecture by the use of a counter example.			
PURE MATHEMATICS: PROOF (2)					
Proof	p3	Understand and be able to use proof by contradiction.	Including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs.		



Chapter 2: Trigonometry	
Understand and use the definition of a radian	
Be able to convert between radians and degrees using $360^\circ = 2\pi$ radians	
Know exact values of $\sin \theta$ and $\cos \theta$ for $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples of these	
Know exact values of $\tan \theta$ for $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and multiples of these	
Know and use the graphs of $\sin \theta, \cos \theta$ and $\tan \theta$, working in radians	
Know and use the formula for arc length, $s = r\theta$, when θ is given in radians	
Know and use the formula for area of a sector, $A = \frac{1}{2}r^2\theta$ when θ is given in radians	
Use the standard small angle approximations of sine, cosine and tangent: $\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{\theta^2}{2}$ and $\tan \theta \approx \theta$ (where θ is in radians)	

Formula Sheet Extract

Small Angle Approximations

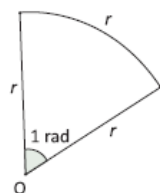
$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta$ where θ is measured in radians

Values to Learn

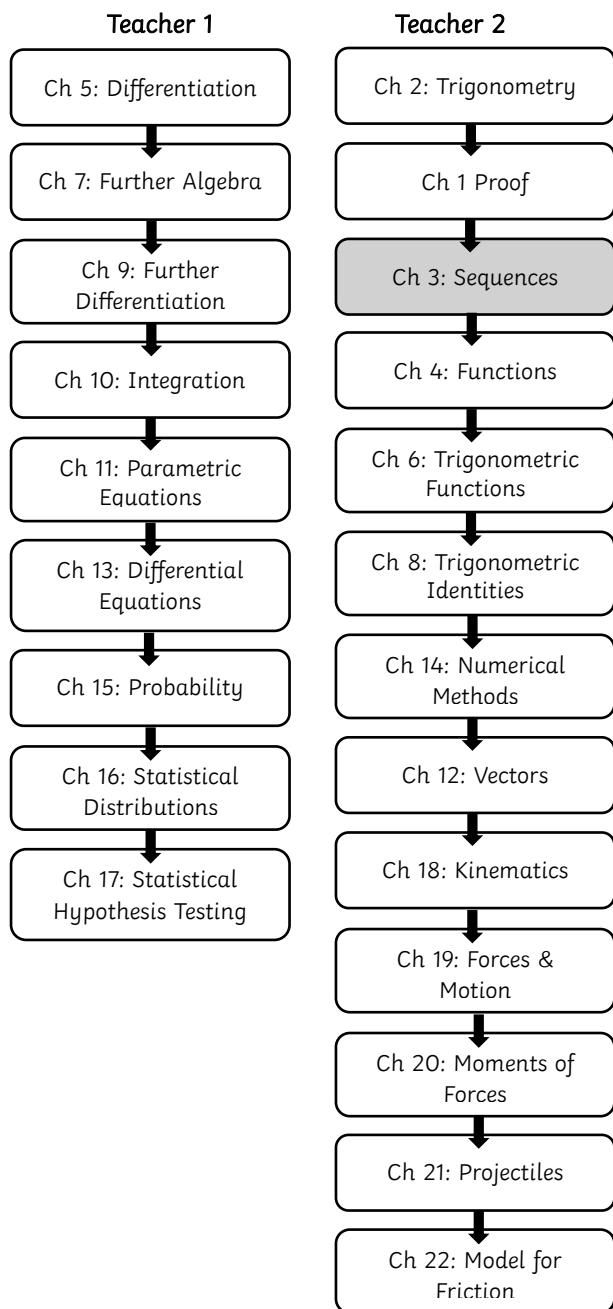
θ°	0°	30°	45°	60°	90°
θ radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: TRIGONOMETRY (2)					
Trig. functions	Mt8	Know and be able to use exact values of $\sin \theta$, $\cos \theta$, $\tan \theta$ for $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and multiples thereof and $\sin \theta$, $\cos \theta$ for $\theta = \frac{\pi}{2}$ and multiples thereof.			
	t9	Understand and use the definitions of the functions arcsin, arccos and arctan, their relationship to sin, cos and tan, their graphs and their ranges and domains.			
Radians	t10	Understand and use the definition of a radian and be able to convert between radians and degrees.			
	t11	Know and be able to find the arc length and area of a sector of a circle, when the angle is given in radians.	The results $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ where θ is measured in radians.		
	t12	Understand and use the standard small angle approximations of sine, cosine and tangent.	$\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{\theta^2}{2}$, $\tan \theta \approx \theta$ where θ is in radians.		

Radians



A radian is the angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.



Chapter 3: Sequences and Series	
Know that a sequence is an ordered set of numbers, $a_1, a_2, a_3, \dots, a_k, \dots, a_n$ where a_k is the general term. It may be finite or infinite.	
Know that a series is the sum of the terms of a sequence: $a_1 + a_2 + a_3 + \dots + a_n = \sum_{k=1}^{k=n} a_k$	
Know that a sequence may be increasing, decreasing, oscillating, periodic, convergent or divergent (or none of these)	
In a periodic sequence, $a_{k+p} = a_k$ for a fixed integer, p , called the period	
In an oscillating sequence, the terms rise above and fall below a middle value	
Be able to generate a sequence using a formula for the k^{th} term, or a recurrence relation of the form $a_{k+1} = f(a_k)$, notation k^{th} term a_k	
Know that an arithmetic sequence (also called an arithmetic progression, AP) has the form $a_{k+1} = a_k + d$ where d is a fixed number called the common difference	
Use arithmetic sequences with first term a and common difference d and know: <ul style="list-style-type: none"> the k^{th} term, $a_k = a + (k - 1)d$ the last term, $l = a + (n - 1)d$ sum of n terms, $S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$ 	
Know that a geometric sequence (also called a geometric progression, GP) has the form $a_{k+1} = ra_k$ where r is a fixed number called the common ratio	
Use geometric sequences with first term a and common ratio r and know: <ul style="list-style-type: none"> the k^{th} term, $a_k = ar^{k-1}$ the last term, $l = ar^{n-1}$ the sum of the first n terms, $S_n = \frac{a(r^n - 1)}{(r - 1)} = \frac{a(1 - r^n)}{(1 - r)}$ 	
Know the condition for an infinite geometric series to be convergent is $-1 < r < 1$ and in this case, the sum of all the terms is given by $S_\infty = \frac{a}{(1 - r)}$	
Be able to use sequences and series in modelling.	

Formula Sheet Extract

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n \{2a + (n - 1)d\}$$

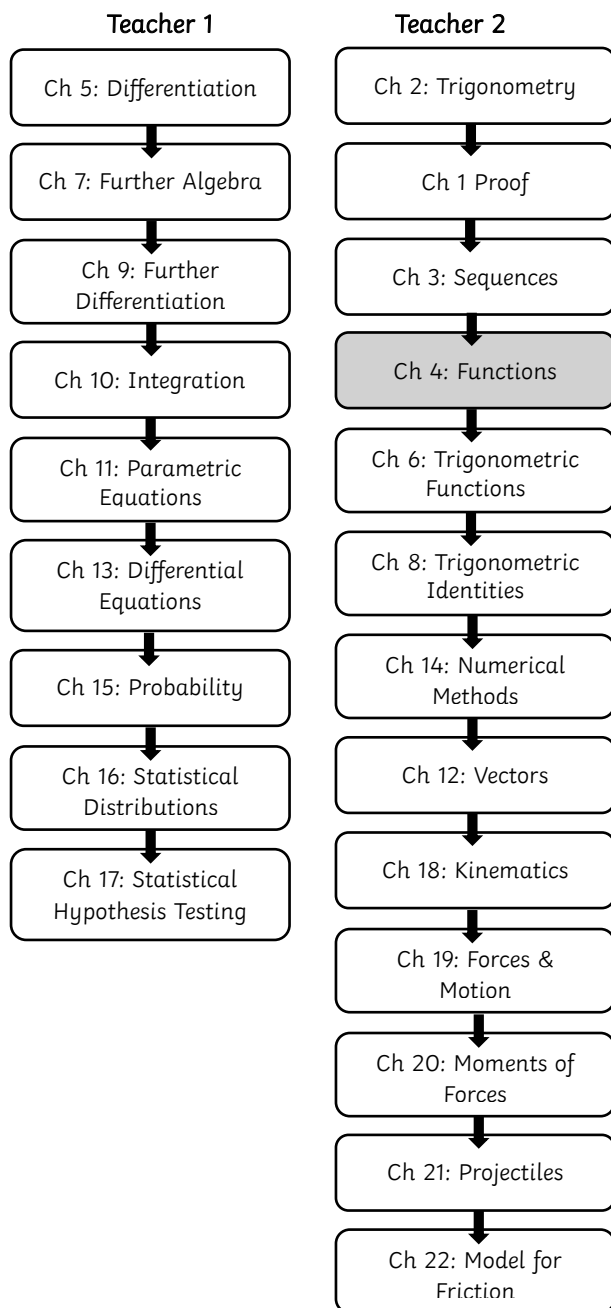
Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: SEQUENCES AND SERIES (2)					
Sequences	Ms6	Know what a sequence of numbers is and the meaning of finite and infinite with reference to sequences.			
	s7	Be able to generate a sequence using a formula for the k^{th} term, or a recurrence relation of the form $a_{k+1} = f(a_k)$.	e.g. $a_k = 2 + 3k$; $a_{k+1} = a_k + 3$ with $a_1 = 5$.	k^{th} term: a_k	
	s8	Know that a series is the sum of consecutive terms of a sequence.	Starting from the first term.		
	s9	Understand and use sigma notation.		$\sum_{r=1}^n r = 1 + 2 + \dots + n$	
	s10	Be able to recognise increasing, decreasing and periodic sequences.			
	s11	Know the difference between convergent and divergent sequences.	Including when using a sequence as a model or when using numerical methods.	Limit to denote the value to which a sequence converges.	Formal tests for convergence.
Arithmetic series	s12	Understand and use arithmetic sequences and series.	The term arithmetic progression (AP) may also be used for an arithmetic sequence.	First term, a Last term, l Common difference, d .	
	s13	Be able to use the standard formulae associated with arithmetic sequences and series.	The n th term, the sum to n terms. Including the sum of the first n natural numbers.	S_n	
Geometric series	s14	Understand and use geometric sequences and series.	The term geometric progression (GP) may also be used for a geometric sequence.	First term, a Common ratio, r .	
	s15	Be able to use the standard formulae associated with geometric sequences and series.	The n th term, the sum to n terms.	S_n	
	s16	Know the condition for a geometric series to be convergent and be able to find its sum to infinity.		$S_{\infty} = \frac{a}{1-r}, r < 1$	
Modelling	s17	Be able to use sequences and series in modelling.			

Learning Journey: Year 13 Maths



Personalised Learning Checklist (PLC): Year 13 Maths

Chapter 4: Functions	
Know that a mapping is any rule connecting inputs (objects) to outputs (images). Mappings can be many-to-one, one-to-many, one-to-one or many-to-many.	
Know that a many-to-one or one-to-one mapping is called a function. It is a mapping where each input value gives exactly one output value.	
Know the domain of a mapping or function is the set of possible inputs (values of x)	
Know the range of a mapping or function is the set of outputs (values of y)	
Work with combined transformations on a graph: form the equation of a transformed graph, sketch the transformed graph or recognise the transformations applied	
Understand and use composite functions, where one function (say g) is applied to the output from another function (say f) to get $g[f(x)]$ or $gf(x)$	
Be able to find the correct domain of (eg) gf given the domains of f and g .	
Know that for an inverse of a function to exist, the function must be one-to-one, and know how to find the inverse function	
Know that the curves of a function and its inverse are reflections of each other in $y = x$, appreciating that the same scale must be used on the x and y axes	
Be able to find the domain and range of an inverse function, $f^{-1}(x)$	
Understand and be able to use the modulus function $ x $, which means the positive value of x	
Be able to draw graphs of the modulus of linear functions, knowing that any part of the graph of the function that is below the x -axis is reflected in the x -axis	
Be able to solve simple inequalities containing a modulus	
Use inequalities of the form $ x - a \leq b$ to express upper and lower bounds, $a \pm b$, for the value of x	
Be able to use functions in modelling, including consideration of limitations and refinements of the models	

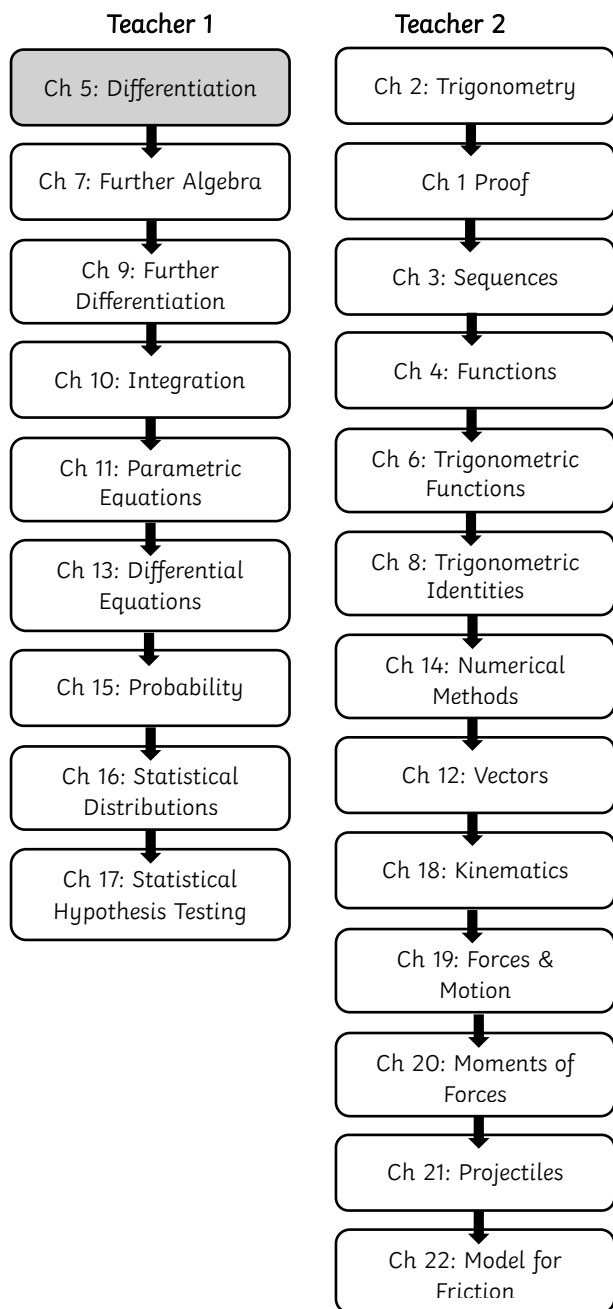
Mathematical Notation

Functions	
$f(x)$	the value of the function f at x
$f: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
gf	the composite function of f and g which is defined by $gf(x) = g(f(x))$

PURE MATHEMATICS: FUNCTIONS (2)					
The language of functions	f3	Understand the definition of a function, and be able to use the associated language.	A function is a mapping from the domain to the range such that for each x in the domain, there is a unique y in the range with $f(x) = y$. The range is the set of all possible values of $f(x)$.	Many-to-one, one-to-one, domain, range. $f : x \rightarrow y$	
	f4	Understand and use composite functions.	Includes finding the correct domain of gf given the domains of f and g .	$gf(x)$	
	f5	Understand and be able to use inverse functions and their graphs. Know the conditions necessary for the inverse of a function to exist and how to find it.	Includes using reflection in the line $y = x$ and finding domain and range of an inverse function. e.g. $\ln x (x > 0)$ is the inverse of e^x .	$f^{-1}(x)$	
The modulus function	f6	Understand and be able to use the modulus function.	Graphs of the modulus of linear functions involving a single modulus sign.		
	f7	Be able to solve simple inequalities containing a modulus sign.	Including the use of inequalities of the form $ x - a \leq b$ to express upper and lower bounds, $a \pm b$, for the value of x .		Inequalities involving more than one modulus sign or modulus of non-linear functions.
Modelling	f8	Be able to use functions in modelling.	Including consideration of limitations and refinements of the models.		

PURE MATHEMATICS: GRAPHS (2)					
Transformations	C8	Understand the effect of combined transformations on a graph and be able to form the equation of the new graph and to sketch it. Be able to recognise the transformations that have been applied to a graph from the graph or its equation.		Vector notation may be used for a translation. $\begin{pmatrix} a \\ b \end{pmatrix}, ai + bj$	

Learning Journey: Year 13 Maths





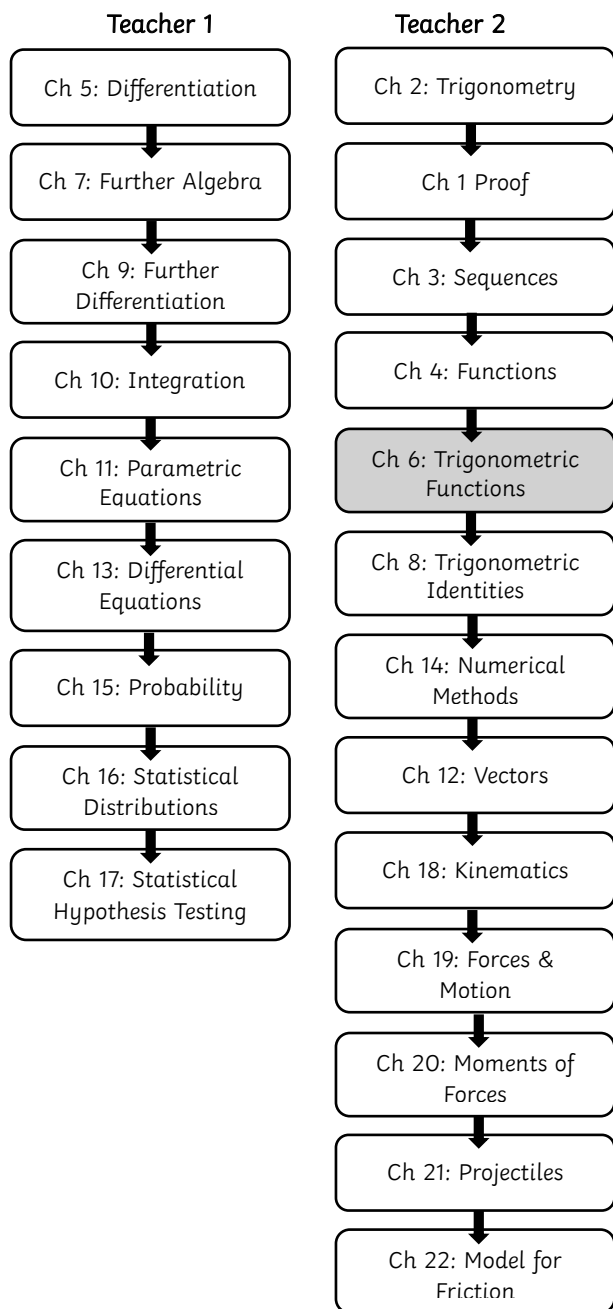
Personalised Learning Checklist (PLC): Year 13 Maths

Chapter 5: Differentiation	
Understand that a section of curve which has increasing gradient is concave upwards and $\frac{d^2y}{dx^2} > 0$	
Understand that a section of curve which has decreasing gradient is concave downwards and $\frac{d^2y}{dx^2} < 0$	
Understand that a point of inflection on a curve is where the curve changes from concave upwards to concave downwards (or vice versa) and hence $\frac{d^2y}{dx^2} = 0$	
Use differentiation to find stationary and non-stationary points of inflection	
Use stationary points of inflection when sketching curves	
Differentiate composite functions using the chain rule $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$	
Differentiate the product of two functions $y = uv$ using the product rule $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$	
Differentiate the quotient of two functions $y = \frac{u}{v}$ using the quotient rule $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$	
Find rates of change using the chain rule, including connected rates of change and differentiation of inverse functions	
Know that $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$	

Formula Sheet Extract

Quotient Rule $y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: CALCULUS (2)					
Applications of differentiation to functions and graphs	c17	Understand that a section of curve which has increasing gradient (and so positive second derivative) is concave upwards. Understand that a section of curve which has decreasing gradient (and so negative second derivative) is concave downwards.	 concave upwards (convex downwards)  concave downwards (convex upwards)	The wording "concave upwards" or "concave downwards" will be used in examination questions.	
	c18	Understand that a point of inflection on a curve is where the curve changes from concave upwards to concave downwards (or vice versa) and hence that the second derivative at a point of inflection is zero. Be able to use differentiation to find stationary and non-stationary points of inflection.	Learners are expected to be able to find and classify points of inflection as stationary or non-stationary. Distinguish between maxima, minima and stationary points of inflection.		
Sketching curves	C9	Be able to use stationary points of inflection when curve sketching.			
Product, quotient and chain rules	c12	Be able to differentiate the product of two functions.	The product rule: $y = uv$, $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ Or $[f(x)g(x)]' = f(x)g'(x) + f'(x)g(x)$		
	c13	Be able to differentiate the quotient of two functions.	$y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ Or $\left[\frac{f(x)}{g(x)} \right]' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$		
	c14	Be able to differentiate composite functions using the chain rule.	$y = f(u), u = g(x), \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ or $\{f[g(x)]\}' = f'[g(x)]g'(x)$		
	c15	Be able to find rates of change using the chain rule, including connected rates of change and differentiation of inverse functions.	$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy} \right)}$		



Chapter 6: Trigonometric Functions	
Know that $\sec \theta = \frac{1}{\cos \theta}$, $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and $\cot \theta = \frac{1}{\tan \theta} (= \frac{\cos \theta}{\sin \theta})$ and understand which angles they are undefined for	
Understand relationships between the graphs of the sin, cos, tan, cosec, sec and cot functions, including domains and ranges	
Use the identities $\sin^2 \theta + \cos^2 \theta \equiv 1$, $\tan^2 \theta + 1 \equiv \sec^2 \theta$, $\cot^2 \theta + 1 \equiv \operatorname{cosec}^2 \theta$	
Solve equations involving sec, cosec and cot	
Understand and use the functions arcsin, arccos and arctan including their relationship to sin, cos and tan, and their domains and ranges	

Values and Formulas to Learn

Function	Restricted Domain (degrees)	Restricted Domain (radians)	Inverse Function
$y = \sin \theta$	$-90 \leq \theta \leq 90$	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$\arcsin \theta$
$y = \cos \theta$	$0 \leq \theta \leq 180$	$0 \leq \theta \leq \pi$	$\arccos \theta$
$y = \tan \theta$	$-90 < \theta < 90$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$\arctan \theta$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

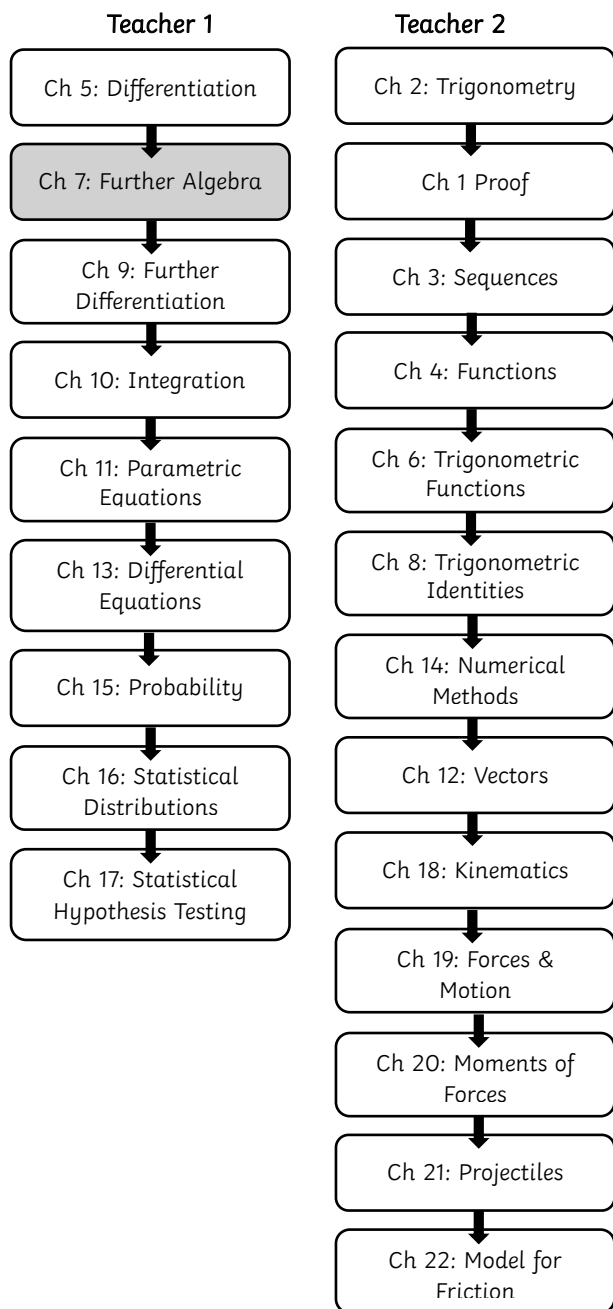
$$\cot \theta = \frac{1}{\tan \theta} (= \frac{\cos \theta}{\sin \theta})$$

$$\sin^2 \theta + \cos^2 \theta \equiv 1$$

$$\tan^2 \theta + 1 \equiv \sec^2 \theta$$

$$\cot^2 \theta + 1 \equiv \operatorname{cosec}^2 \theta$$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: TRIGONOMETRY (2)					
Secant, cosecant and cotangent	t13	Understand and use the definitions of the sec, cosec and cot functions.	Including knowledge of the angles for which they are undefined.		
	t14	Understand relationships between the graphs of the sin, cos, tan, cosec, sec and cot functions.	Including domains and ranges.		
	t15	Understand and use the relationships $\tan^2\theta + 1 = \sec^2\theta$ and $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$.			
Equations	t19	Use trigonometric identities, relationships and definitions in solving equations.			
Proofs and problems	t20	Construct proofs involving trigonometric functions and identities.			
	t21	Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.	The argument of the trigonometric functions is not restricted to angles.		



Chapter 7: Further Algebra	
Use the binomial expansion of $(1+x)^n$ where n is any rational number for $ x < 1$ when n is not a positive integer	
Write $(a+bx)^n$ in the form $a^n(1+\frac{bx}{a})^n$ and hence expand $(a+bx)^n$ for $ \frac{bx}{a} < 1$ when n is not a positive integer	
Use binomial expansions with n rational to find polynomials which approximate $(a+bx)^n$, including finding approximations to rational powers of numbers	
Know the condition for the binomial expansion to be valid	
Simplify rational expressions by factorising and cancelling, or by any correct method of algebraic division	
Express algebraic fractions as partial fractions (fractions with constant or linear numerators and denominators up to three linear terms, including squared linear terms in denominator)	

Formula Sheet Extract

Binomial series

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

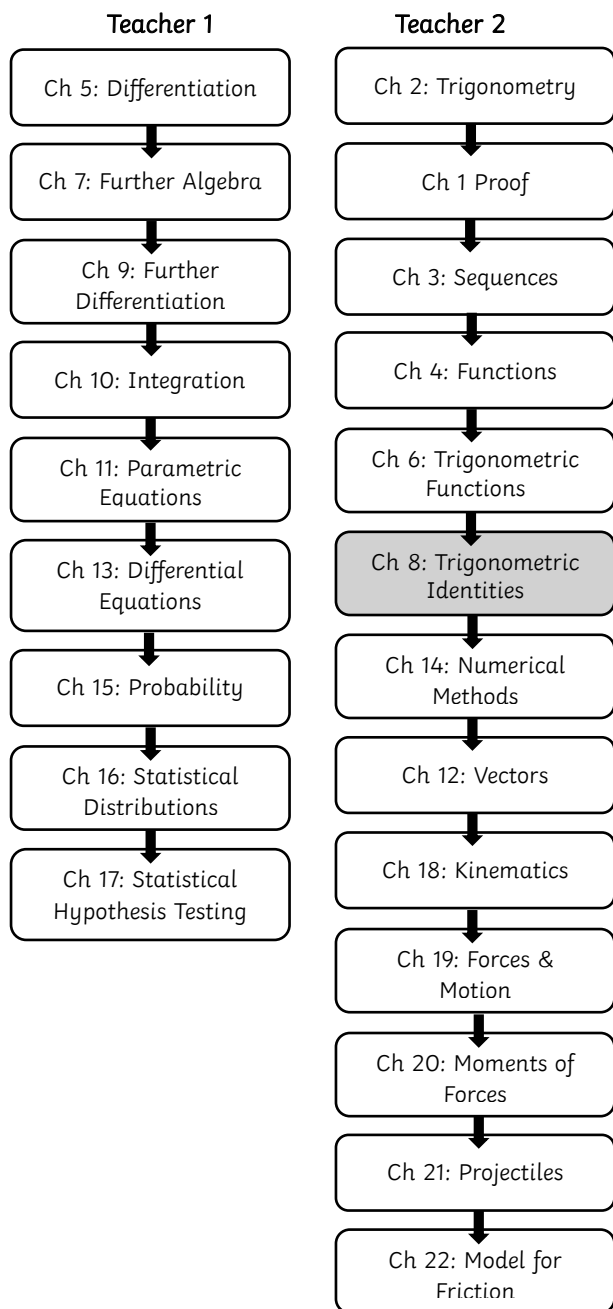
Partial Fraction Formats to Learn

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d} \quad \text{or} \quad \frac{px+q}{(ax+b)(cx+d)(ex+f)} = \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{ex+f}$$

$$\frac{px+q}{(ax+b)(cx+d)^2} = \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$$

Specification: OCR Mathematics B (MEI) H640

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: SEQUENCES AND SERIES (1)					
Binomial expansions	Ms1	Understand and use the binomial expansion of $(a + bx)^n$ where n is a positive integer.			
	s2	Know the notations $n!$ and ${}_n C_r$ and that ${}_n C_r$ is the number of ways of selecting r distinct objects from n .	The meaning of the term factorial. n a positive integer. Link to binomial probabilities.	${}_n C_r = \frac{n!}{r!(n-r)!}$ $n! = 1.2.3...n$ ${}_n C_0 = {}_n C_n = 1$ $0! = 1$ ${}_n C_r = \binom{n}{r}$	${}_n C_r$ will only be used in the context of binomial expansions and binomial probabilities.
PURE MATHEMATICS: SEQUENCES AND SERIES (2)					
Binomial expansions	s3	Use the binomial expansion of $(1 + x)^n$ where n is any rational number.	For $ x < 1$ when n is not a positive integer.		General term.
	s4	Be able to write $(a + bx)^n$ in the form $a^n \left(1 + \frac{bx}{a}\right)^n$ and hence expand $(a + bx)^n$.	$\left \frac{bx}{a}\right < 1$ when n is not a positive integer.		Proof of convergence.
	s5	Be able to use binomial expansions with n rational to find polynomials which approximate $(a + bx)^n$.	Includes finding approximations to rational powers of numbers.		
PURE MATHEMATICS: ALGEBRA (2)					
Partial fractions	a15	Be able to express algebraic fractions as partial fractions.	Fractions with constant or linear numerators and denominators up to three linear terms. Includes squared linear terms in denominator.		Fractions with a quadratic or cubic which cannot be factorised in the denominator.
Rational expressions	a16	Be able to simplify rational expressions.	Including factorising, cancelling and simple algebraic division. Any correct method of algebraic division may be used.		Division by non-linear expressions.



Chapter 8: Trigonometric Identities	
Understand and use the compound angle identities for $\sin(\theta \pm \alpha)$, $\cos(\theta \pm \alpha)$, $\tan(\theta \pm \alpha)$	
Understand the geometric proofs for compound angle identities (a starting point will be given – proof using de Moivre's theorem will not be accepted)	
Know and use the identities for double angles, deriving them from the formulas for $\sin(\theta + \alpha)$, $\cos(\theta + \alpha)$, $\tan(\theta + \alpha)$ $\sin 2\theta = 2 \sin \theta \cos \theta$, $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$, $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$	
Know the alternative versions of the formula for $\cos 2\theta$: $\cos 2\theta = 2\cos^2 \theta - 1$ or $\cos 2\theta = 1 - 2\sin^2 \theta$	
Write $a \cos \theta + b \sin \theta$ in the equivalent forms $r \sin(\theta \pm \alpha)$ and $r \cos(\theta \pm \alpha)$	
Use the form $r \sin(\theta \pm \alpha)$ and $r \cos(\theta \pm \alpha)$ to find maximum or minimum values, solve equations and sketch the graph of a function	
Use trigonometric identities, relationships and definitions in solving equations	
Construct proofs involving trigonometric functions and identities	
Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces	

Formula Sheet Extract

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

Formulas to Learn

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

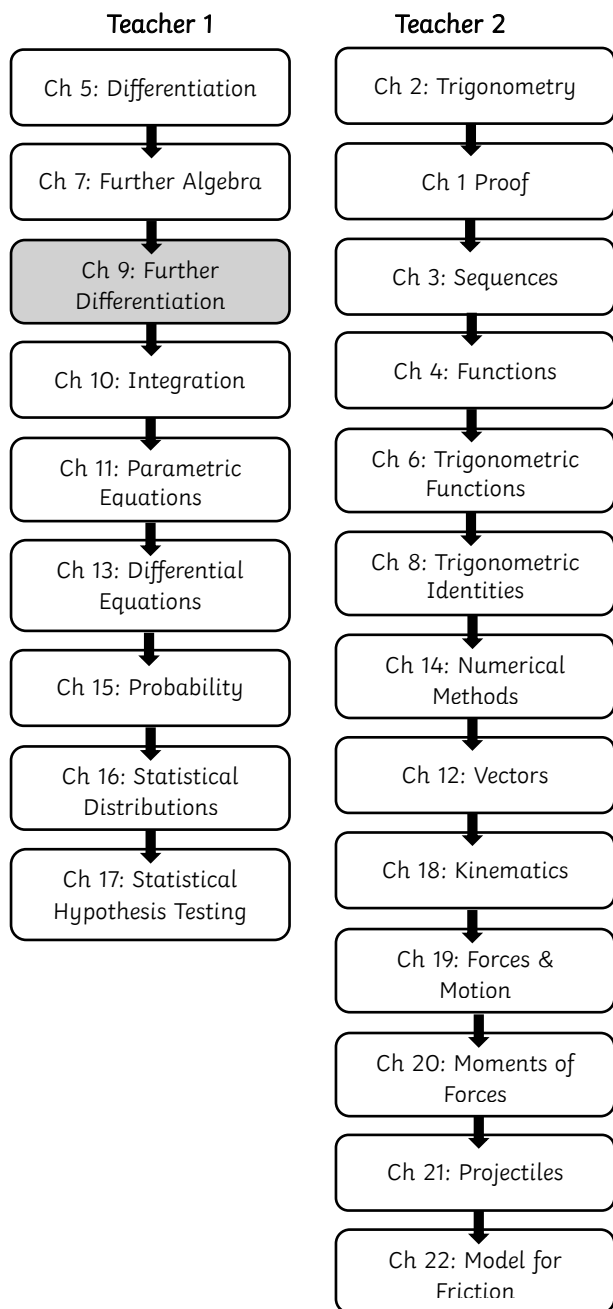
$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: TRIGONOMETRY (2)					
Compound angle formulae	Mt16	Understand and use the identities for $\sin(\theta \pm \phi)$, $\cos(\theta \pm \phi)$, $\tan(\theta \pm \phi)$.	Includes understanding geometric proofs. The starting point for the proof will be given.		Proofs using de Moivre's theorem will not be accepted.
	t17	Know and use identities for $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$.	Includes understanding derivations from $\sin(\theta + \phi)$, $\cos(\theta + \phi)$, $\tan(\theta + \phi)$. $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $\cos 2\theta = 2 \cos^2 \theta - 1$ $\cos 2\theta = 1 - 2 \sin^2 \theta$		
	t18	Understand and use expressions for $a \cos \theta \pm b \sin \theta$ in the equivalent forms $R \sin(\theta \pm \alpha)$ and $R \cos(\theta \pm \alpha)$.	Includes sketching the graph of the function, finding its maximum and minimum values and solving equations.		
Equations	t19	Use trigonometric identities, relationships and definitions in solving equations.			
Proofs and problems	t20	Construct proofs involving trigonometric functions and identities.			
	t21	Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.	The argument of the trigonometric functions is not restricted to angles.		



Chapter 9: Further Differentiation	
Differentiate e^{kx} , a^{kx} and $\ln x$, including related sums, differences and constant multiples.	
Differentiate $\sin kx$, $\cos kx$ and $\tan kx$ where x is in radians, including their constant multiples, sums and differences	
Differentiation from first principles of $\sin x$ and $\cos x$	
Use the chain rule, product rule and quotient rule to differentiate functions involving the functions above	
Differentiate functions defined implicitly	
Differentiate inverse functions using implicit differentiation or using $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$	

Formula Sheet Extract

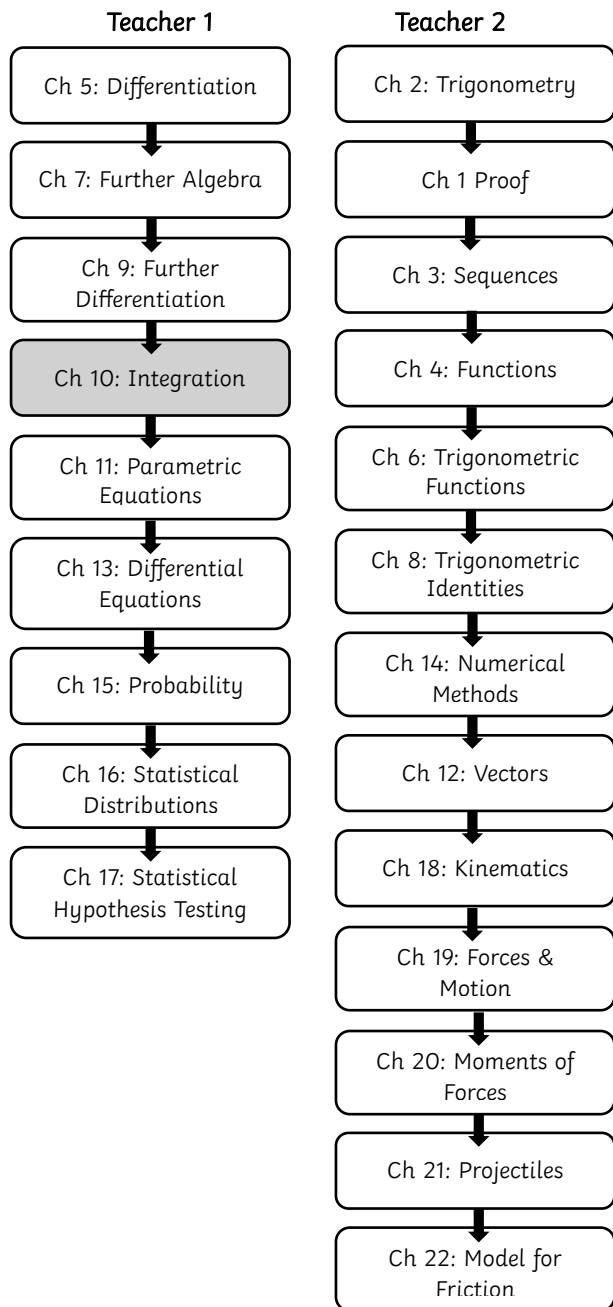
Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\text{cosec}^2 x$
$\text{cosec } x$	$-\text{cosec } x \cot x$

Results to Learn

$f(x)$	$f'(x)$
$\ln kx$	$\frac{1}{x}$
e^x or e^{kx}	e^x or ke^{kx}
a^x or a^{kx}	$a^x \ln a$ or $ka^{kx} \ln a$
$\sin x$ or $\sin kx$	$\cos x$ or $k \cos kx$
$\cos x$ or $\cos kx$	$-\sin x$ or $-k \sin kx$
$f(y)$	$f'(y) \times \frac{dy}{dx}$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: CALCULUS (2)					
Differentiation of functions	Mc10	Be able to differentiate e^{kx} , a^{kx} and $\ln x$.	Including related sums, differences and constant multiples.		
	c11	Be able to differentiate the trigonometrical functions: $\sin kx$; $\cos kx$; $\tan kx$ for x in radians.	Including their constant multiples, sums and differences. Differentiation from first principles for $\sin x$ and $\cos x$.		
Implicit differentiation	c16	Be able to differentiate a function or relation defined implicitly.	e.g. $(x + y)^2 = 2x$.		Second and higher derivatives.



Chapter 10: Integration	
Understand integration as the limit of a sum	$\lim_{\delta x \rightarrow 0} \sum_a^b f(x) \delta x = \int_a^b f(x) dx$
Use integration to find the area between a curve and the x -axis or y -axis, including integrating with respect to y	
Use integration to find the area between two curves	
Use integration by substitution where the process is the reverse of the chain rule	
Integrate e^{kx} , $\sin kx$, $\cos kx$ and related sums, differences and constant multiples	
Integrate $\frac{1}{x}$ and integrals of the form $\frac{f'(x)}{f(x)}$	
Use partial fractions in integration	
Use integration by substitution in other cases, finding a suitable substitution	
Use the method of integration by parts, including cases where more than one application may be required	
Use integration by parts to integrate $\ln x$ by treating it as $1 \times \ln x$	

Formula Sheet Extract

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

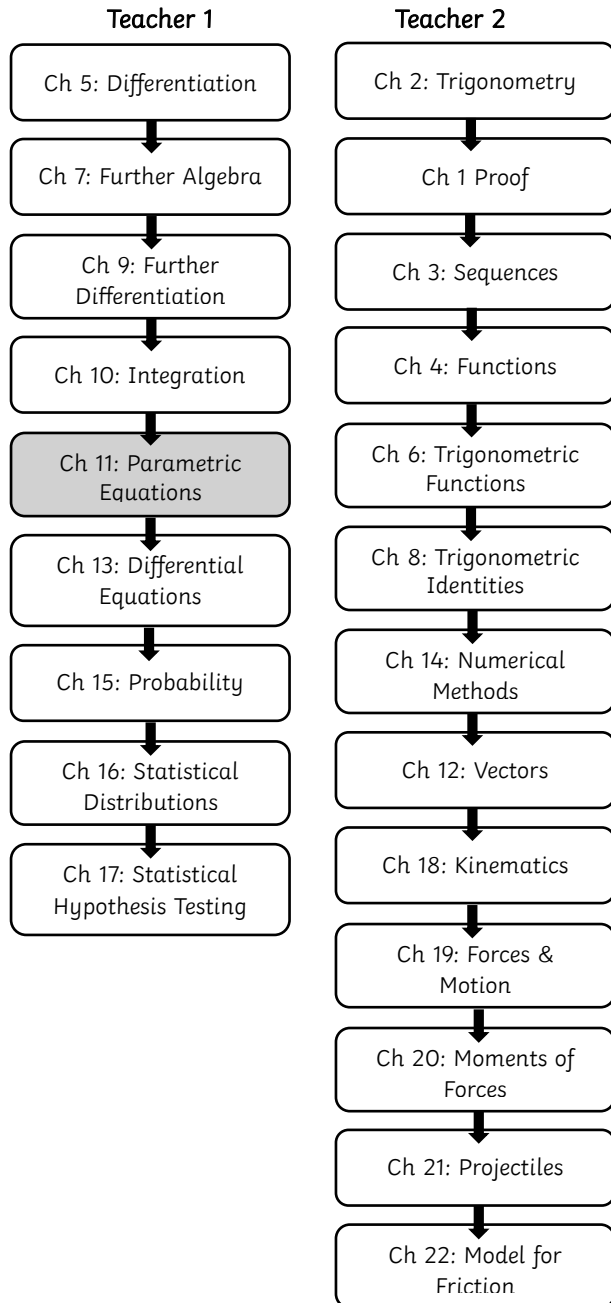
Integration by parts $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Results to Learn

$f(x)$	$\int f(x) dx$
e^x or e^{kx}	$e^x + c$ or $\frac{1}{k}e^{kx} + c$
$\sin x$ or $\sin kx$	$-\cos x + c$ or $-\frac{1}{k}\cos kx + c$
$\cos x$ or $\cos kx$	$\sin x + c$ or $\frac{1}{k}\sin kx + c$
$\sec^2 x$ or $\sec^2 kx$	$\tan x + c$ or $\frac{1}{k}\tan kx + c$
$\frac{1}{x}$	$\ln x + c$ or $\ln kx $ where $k > 0$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: CALCULUS (2)					
Integration as inverse of differentiation	Mc24	Be able to integrate e^{kx} , $\frac{1}{x}$, $\sin kx$, $\cos kx$ and related sums, differences and constant multiples.	$\int \frac{1}{x} dx = \ln x + c$, $x \neq 0$ x in radians for trigonometrical integrals.		Integrals involving inverse trigonometrical functions.
Integration to find area under a curve	c25	Understand integration as the limit of a sum.	Know that $\lim_{\delta x \rightarrow 0} \sum_a^b f(x) \delta x = \int_a^b f(x) dx$		
	c26	Be able to use integration to find the area between two curves.	Learners should also be able to find the area between a curve and the y -axis, including integrating with respect to y .		
Integration by substitution	c27	Be able to use integration by substitution in cases where the process is the reverse of the chain rule (including finding a suitable substitution).	e.g. $(1 + 2x)^8$, $x(1 + x^2)^8$, xe^{x^2} , $\frac{1}{2x + 3}$ Learners can recognise the integral, they need not show all the working for the substitution.		
	c28	Be able to use integration by substitution in other cases.	Learners will be expected to find a suitable substitution in simple cases e.g. $\frac{x}{(x + 1)^3}$.		Integrals requiring more than one substitution before they can be integrated.
Integration by parts	c29	Be able to use the method of integration by parts in simple cases.	Includes cases where the process is the reverse of the product rule. e.g. xe^x . More than one application of the method may be required. Includes being able to apply integration by parts to $\ln x$.		Reduction formulae.
Partial fractions	c30	Be able to integrate using partial fractions that are linear in the denominator.			

Learning Journey: Year 13 Maths



Personalised Learning Checklist (PLC): Year 13 Maths

Chapter 11: Parametric Equations	
Understand the meaning of the term parameter and parametric equations	
Convert between Cartesian and parametric forms of equations	
Use the equation of a circle written in parametric form	
Use parametric differentiation to find the gradient function of a curve defined parametrically	
Use parametric equations in modelling contexts such as kinematics and projectiles in mechanics, including modelling with a parameter with a restricted domain.	

Results to Learn

Equation of a circle with centre (0, 0) and radius r:

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Equation of a circle with centre (a, b) and radius r:

$$x = a + r \cos \theta$$

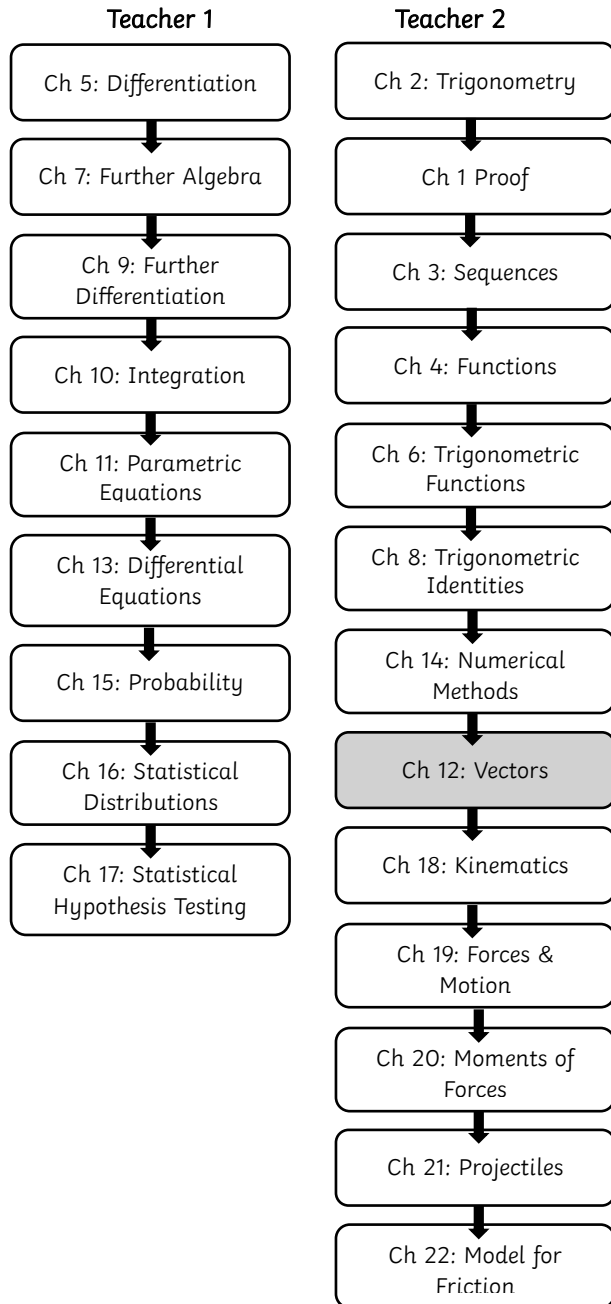
$$y = b + r \sin \theta$$

Parametric Differentiation:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad \text{provided } \frac{dx}{dt} \neq 0$$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: COORDINATE GEOMETRY (2)					
Parametric equations	g12	Understand the meaning of the terms parameter and parametric equations.			
	g13	Be able to convert between cartesian and parametric forms of equations.	When converting from cartesian to parametric form, guidance will be given as to the choice of parameter.		
	g14	Understand and use the equation of a circle written in parametric form.			
	g15	Be able to find the gradient of a curve defined in terms of a parameter by differentiation.	$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$		Second and higher derivatives
	g16	Be able to use parametric equations in modelling.	Contexts include kinematics and projectiles in mechanics. Including modelling with a parameter with a restricted domain.		

Learning Journey: Year 13 Maths



Personalised Learning Checklist (PLC): Year 13 Maths

Chapter 12: Vectors	
Understand the term scalar and vector	
Understand vectors in two and three dimensions, and express them <ul style="list-style-type: none"> in magnitude-direction form for two dimensions using \mathbf{i}, \mathbf{j}, and \mathbf{k} vectors using column vectors using \overrightarrow{OP} notation 	
Understand equal vectors	
Understand the link between the coordinates of a point and its position vector	
Multiply a vector by a scalar	
Add and subtract vectors	
Find a unit vector in the direction of a given vector	
Understand that vectors are parallel when one is a scalar multiple of the other	
Use vectors in geometry problems	

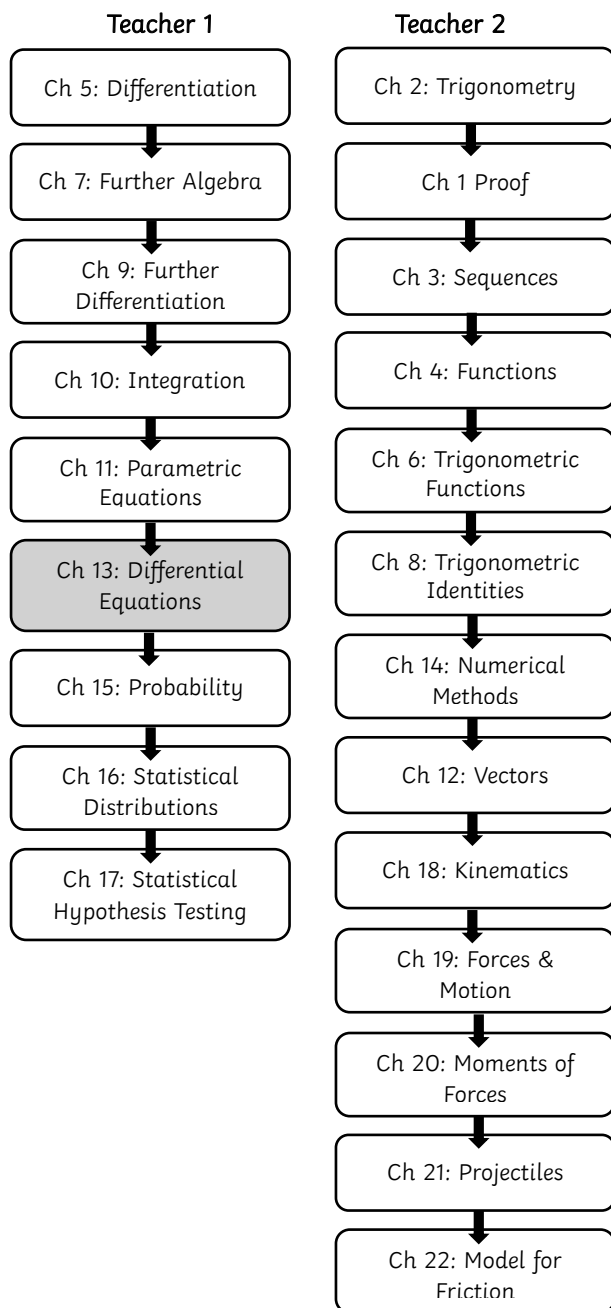
Formulas to Learn

The magnitude of the vector $\mathbf{r} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is $|\mathbf{r}| = \sqrt{a^2 + b^2 + c^2}$

A unit vector in the same direction as $\mathbf{r} = a\mathbf{i} + b\mathbf{j}$ is $\hat{\mathbf{r}} = \frac{a}{\sqrt{a^2+b^2}}\mathbf{i} + \frac{b}{\sqrt{a^2+b^2}}\mathbf{j}$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: VECTORS (1)					
General vectors	Mv1	Understand the language of vectors in two dimensions.	Scalar, vector, modulus, magnitude, direction, position vector, unit vector, cartesian components, equal vectors, parallel vectors, collinear.	Vectors printed in bold . Unit vectors $\mathbf{i}, \mathbf{j}, \hat{\mathbf{r}}$ The magnitude of the vector \mathbf{a} is written $ \mathbf{a} $ or a . $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$	
	v2	Be able to add and subtract vectors using a diagram or algebraically, multiply a vector by a scalar, and express a vector as a combination of others.	Geometrical interpretation. Includes general vectors not expressed in component form.		
	v3	Be able to calculate the magnitude and direction of a vector and convert between component form and magnitude-direction form.		Magnitude-direction	
Position vectors	v4	Understand and use position vectors.	Including interpreting components of a position vector as the cartesian coordinates of the point. $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$	\overrightarrow{OB} or \mathbf{b} . $\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix}$	
	v5	Be able to calculate the distance between two points represented by position vectors.			
Using vectors	v6	Be able to use vectors to solve problems in pure mathematics and in context, including problems involving forces.	Includes interpreting the sum of vectors representing forces as the resultant force.		
PURE MATHEMATICS: VECTORS (2)					
General vectors	Mv7	Understand the language of vectors in three dimensions.	Extend the work of Mv2 to Mv 6 to include vectors in three dimensions.	Unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}, \hat{\mathbf{r}}$ $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$	

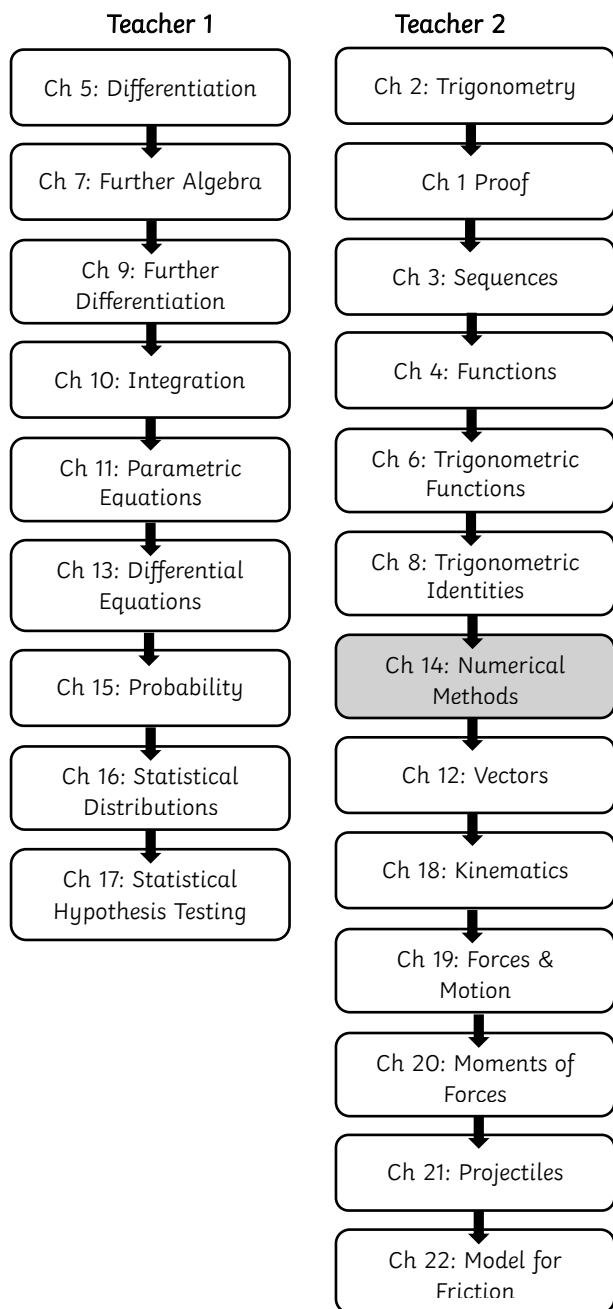
Learning Journey: Year 13 Maths



Personalised Learning Checklist (PLC): Year 13 Maths

Chapter 13: Differential Equations	
Know that differential equations are used to model rates of change	
Know that $\frac{dy}{dx}$ is the rate of change of y with respect to x	
Know that when 'with respect to' is missing, it is assumed to be with respect to time	
Know that a first order differential equation involves (eg) $\frac{dy}{dx}$	
Formulate first order differential equations using information about rates of change	
Use contexts such as kinematics, population growth and modelling the relationship between price and demand	
Solve a simple differential equation using direct integration	
Find a general solution of a first order differential equation by separating variables	
Know that a general solution has an arbitrary constant and gives a family of curves	
Find a particular solution of a first order differential equation by separating variables and using the additional information in the question	
Know that a particular solution is a specific curve from the general solution	
Solve equations which need to be factorised using a common factor before variables can be separated	
Solve problems using differential equations, interpret the solution and identify limitations of the solution	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: CALCULUS (2)					
Differential equations	c31	Be able to formulate first order differential equations using information about rates of change.	Contexts may include kinematics, population growth and modelling the relationship between price and demand.		
	c32	Be able to find general or particular solutions of first order differential equations analytically by separating variables.	Equations may need to be factorised using a common factor before variables can be separated.		
	c33	Be able to interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution.	Includes links to kinematics.		



Chapter 14: Numerical Methods	
Know that if $f(x)$ is a continuous function and $f(a)$ and $f(b)$ have opposite signs, there will be at least one root in the interval $[a, b]$	
Use a change of sign method to find an interval $[a, b]$ in which a root lies	
Know when a change of sign method may fail, eg <ul style="list-style-type: none"> when $y = f(x)$ touches the x-axis when $y = f(x)$ has a vertical asymptote (discontinuous) when there are several roots in the interval 	
Use fixed point iteration to solve $f(x) = 0$ by rearranging it into the form $x = g(x)$ and using the iteration $x_{n+1} = g(x_n)$	
Draw the associated cobweb or staircase diagram for the rearrangement method	
Know that different rearrangements can be used, but they will only converge to the required root at $x = a$ if $-1 < g'(a) < 1$	
Know that the rearrangement methods fails if it diverges, or if it converges to a different root and not the required root	
Use fixed point iteration to solve $f(x) = 0$ with the Newton-Raphson method using the iteration $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	
Represent the Newton-Raphson process graphically and understand how it can fail	
Find an approximate value of a definite integral using the trapezium rule $\int_a^b y \, dx \approx \frac{1}{2} h ((y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}))$	
Know that the trapezium rule gives an over-estimate if the curve is concave upwards, or an under-estimate if it is concave downwards	
Use the sum of a series of rectangles to find an upper bound and lower bound for the area under a curve	
Use numerical methods to solve problems in context	

Formula sheet extract

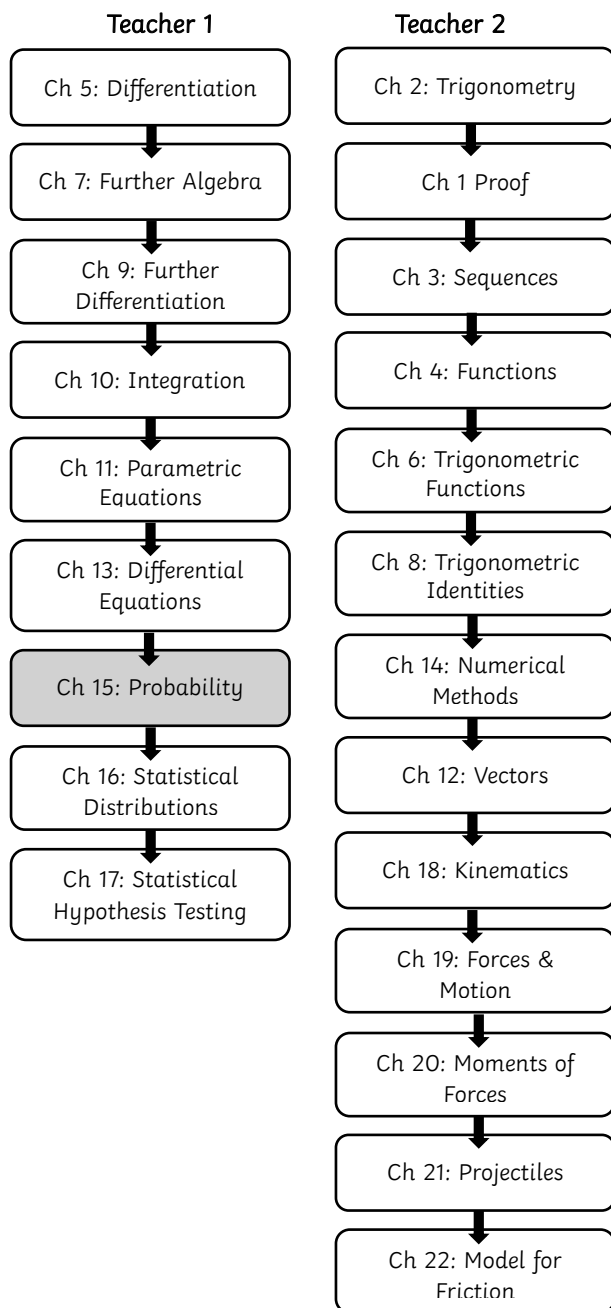
Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2} h \{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
PURE MATHEMATICS: NUMERICAL METHODS (2)					
Solution of equations	Me1	Be able to locate the roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is sufficiently well-behaved.	Finding an interval in which a root lies. This is often used as a preliminary step to find a starting value for the methods in Me3 and Me4.		
	e2	Be aware of circumstances under which change of sign methods may fail.	e.g. when the curve of $y = f(x)$ touches the x -axis. e.g. when the curve of $y = f(x)$ has a vertical asymptote. e.g. there may be several roots in the interval.		
	e3	Be able to carry out a fixed point iteration after rearranging an equation into the form $x = g(x)$ and be able to draw associated staircase and cobweb diagrams.	e.g. write $x^3 - x - 4 = 0$ as $x = \sqrt[3]{x + 4}$ and use the iteration $x_{n+1} = \sqrt[3]{x_n + 4}$ with an appropriate starting value. Includes use of ANS key on calculator.	iteration, iterate	
	e4	Be able to use the Newton-Raphson method to find a root of an equation and represent the process on a graph.			
	e5	Understand that not all iterations converge to a particular root of an equation.	Know how Newton-Raphson and fixed point iteration can fail and be able to show this graphically.		
Integration	Mc34	Be able to find an approximate value of a definite integral using the trapezium rule, and decide whether it is an over- or an under-estimate.	In an interval where the curve is either concave upwards or concave downwards.	Number of strips.	
	c35	Use the sum of a series of rectangles to find an upper and/or lower bound on the area under a curve.			
Problem solving	Me6	Use numerical methods to solve problems in context.			

Learning Journey: Year 13 Maths



Personalised Learning Checklist (PLC): Year 13 Maths

Chapter 15: Probability	
Understand and use mutually exclusive events and independent events and associated notation and definitions	
Know that $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$ for mutually exclusive events	
Use Venn diagrams with up to three events in probability calculations	
Calculate probabilities for two events which are not mutually exclusive	
Know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for non-mutually exclusive events	
Calculate conditional probabilities by formula, from tree diagrams, two-way tables, Venn diagrams or sample space diagrams.	
Know the probability of A occurring given that B has already occurred is given by the formula $P(A B) = \frac{P(A \cap B)}{P(B)}$	
Know the probability of event A and then event B occurring, in that order, is given by $P(A) \times P(B A)$	
Know that if A and B are independent, $P(A \cap B) = P(A) \times P(B)$	
Know that if A and B are independent, $P(B A) = P(B A') = P(B)$	
Know that $P(B A) = P(B) \Leftrightarrow B$ and A are independent	

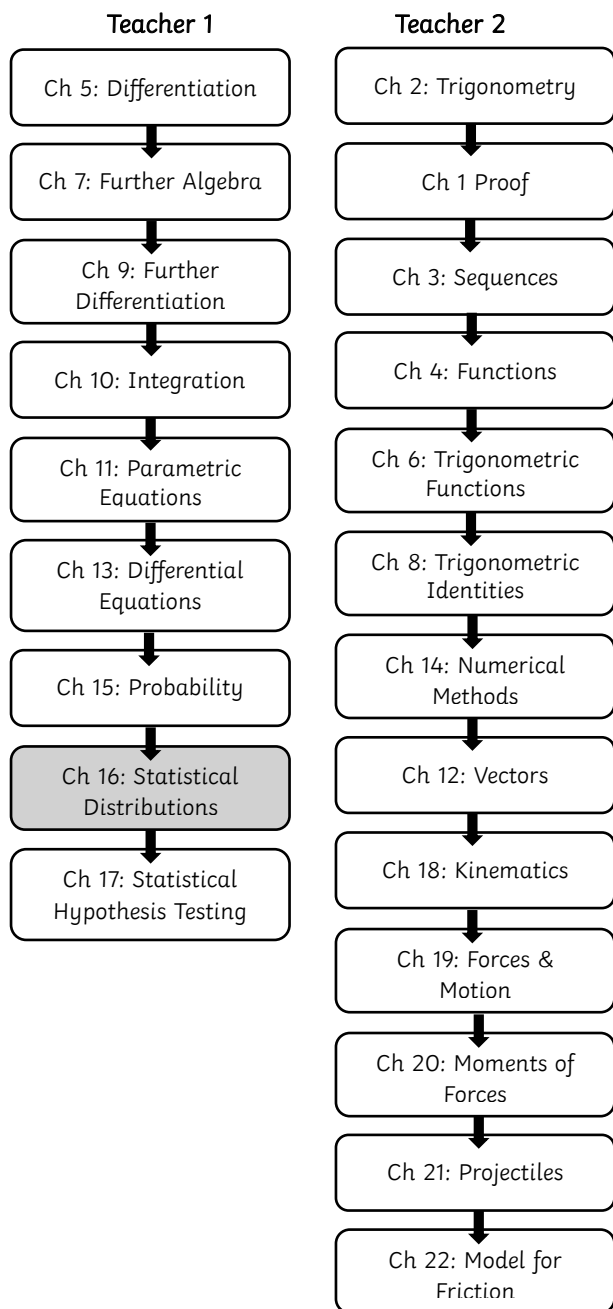
Formula Sheet Extract

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS: PROBABILITY (2)					
Probability of two or more events	u4	Understand and use mutually exclusive events and independent events and associated notation and definitions.	For mutually exclusive events $P(A \cap B) = 0$ for any pair of events.		
	u5	Be able to use Venn diagrams to assist in the calculations of probabilities. Know how to calculate probabilities for two events which are not mutually exclusive.	Venn diagrams for up to three events. Learners should understand the relation: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.		Probability of a general or infinite number of events. Formal proofs.
Conditional probability	u6	Be able to calculate conditional probabilities by formula, from tree diagrams, two-way tables, Venn diagrams or sample space diagrams.	$P(A B) = \frac{P(A \cap B)}{P(B)}$	$P(A B)$	Finding reverse conditional probability i.e. calculating $P(B A)$ given $P(A B)$ and additional information.
	u7	Know that $P(B A) = P(B) \Leftrightarrow B$ and A are independent.	In this case $P(A \cap B) = P(A).P(B)$.		



Chapter 16: Statistical Distributions	
Recognise situations that give rise to a binomial distribution, and express a binomial model in the form $B(n, p)$	
Calculate probabilities, mean and expected frequencies associated with using the binomial distribution	
Use a probability function given algebraically or in a table	
Use the discrete uniform distribution	
Use the Normal distribution $N(\mu, \sigma^2)$ as a model and recognise when it may not be appropriate	
Know the shape of the Normal curve and understand that histograms from increasingly large samples from a Normal distribution tend to the Normal curve.	
Know that linear transformation of a Normal variable gives another Normal variable and know how the mean and standard deviation are affected.	
Standardise a Normal variable using $z = \frac{x - \mu}{\sigma}$ to give $N(0, 1)$ with mean 0 and standard deviation = variance = 1	
Know that the line of symmetry of the Normal curve is located at the mean and the points of inflection are located one standard deviation away from the mean	
Calculate and use probabilities from a Normal distribution, knowing that area under the curve represents probability,	
Understand how and why a continuity correction is used when using a Normal distribution as a model for a distribution of discrete data.	
Recognise from the binomial distribution shape when it can be approximated by a Normal distribution $N(np, npq)$, with n large and p not close to 0 or 1	
Model with probability distributions, recognising when the binomial or Normal model may not be appropriate.	

Formula Sheet Extracts

The Binomial Distribution

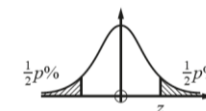
If $X \sim B(n, p)$ then $P(X = r) = {}^n C_r p^r q^{n-r}$ where $q = 1 - p$
 Mean of X is np

Hypothesis test for the mean of a Normal distribution

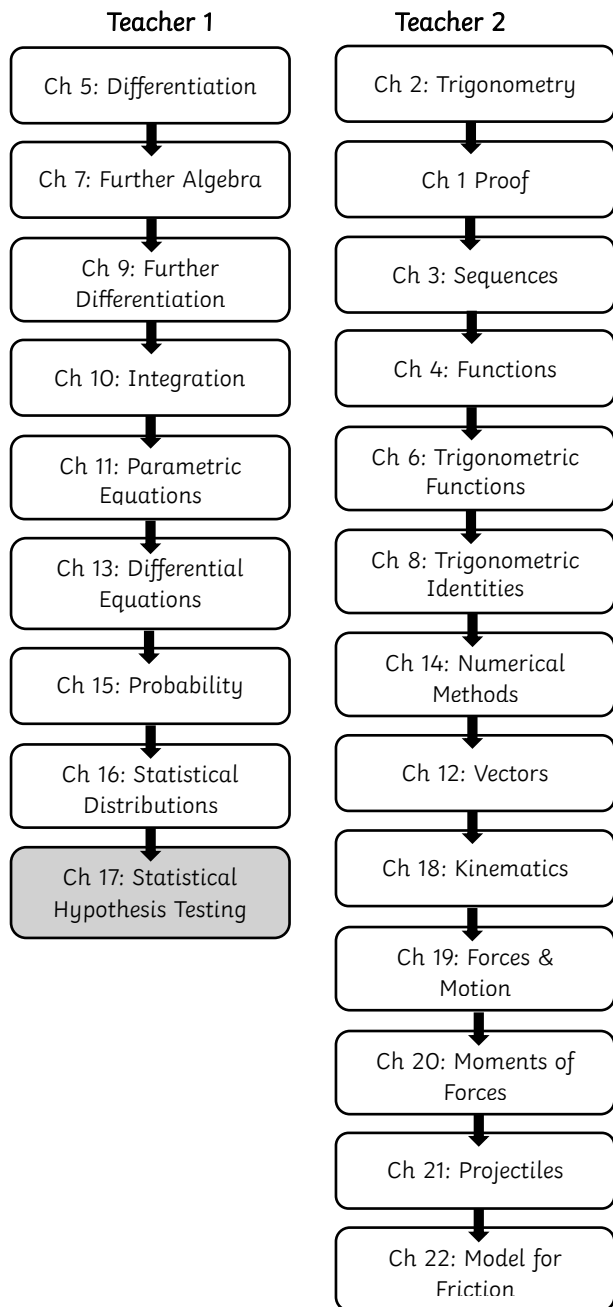
If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS: PROBABILITY DISTRIBUTIONS (2)					
Normal distribution	MR8	Be able to use the Normal distribution as a model.	Includes recognising when a Normal distribution may not be appropriate. Understand how and why a continuity correction is used when using a Normal distribution as a model for a distribution of discrete data. Recognise from the shape of the distribution when a binomial distribution can be approximated by a Normal distribution.	$X \sim N(\mu, \sigma^2)$	Knowing conditions for Normal approximation to binomial.
	R9	Know the shape of the Normal curve and understand that histograms from increasingly large samples from a Normal distribution tend to the Normal curve.	Includes understanding that the area under the Normal curve represents probability.		
	R10	Know that linear transformation of a Normal variable gives another Normal variable and know how the mean and standard deviation are affected. Be able to standardise a Normal variable.	$y_i = a + bx_i \Rightarrow \bar{y} = a + b\bar{x}, s_y^2 = b^2s_x^2$	Standard Normal $Z \sim N(0, 1)$ $Z = \frac{X - \mu}{\sigma}$	Proof
	R11	Know that the line of symmetry of the Normal curve is located at the mean and the points of inflection are located one standard deviation away from the mean.			
	R12	Be able to calculate and use probabilities from a Normal distribution.	Including use of calculator functions.		
Modelling with probability	R13	Be able to model with probability and probability distributions, including recognising when the binomial or Normal model may not be appropriate.	Including critiquing assumptions made and the likely effect of more realistic assumptions.		
Mean and variance of a Normal distribution					
The Normal distribution is a probability model; its mean and variance are calculated using techniques beyond the scope of A Level Mathematics. At this level, students should understand the mean and variance of a Normal distribution as the limiting values from calculating the mean and variance of increasingly large samples from a Normal distribution.					



Chapter 17: Statistical Hypothesis Testing	
Know that random samples of size n from $X \sim N(\mu, \sigma^2)$ have the sample mean Normally distributed with mean μ and variance $\frac{\sigma^2}{n}$	
Carry out a hypothesis test for a single mean using the Normal distribution, using either a p-value or a critical region, and interpret the results in context when <ul style="list-style-type: none"> The population variance is known The population variance is unknown but the sample size is large 	
Identify the critical and acceptance regions for a hypothesis test	
Understand the meaning of correlation, association and rank correlation <ul style="list-style-type: none"> correlation is a measure of how close data points lie to a straight line rank correlation coefficient measures the correlation between the data ranks rather than actual data values association refers to a more general relationship between the variables 	
Use a given correlation coefficient for a sample to make an inference about correlation or association in the population for given p-value or critical value.	
Know that correlation does not imply causation	

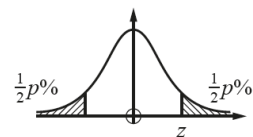
Formula Sheet Extract

Hypothesis test for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

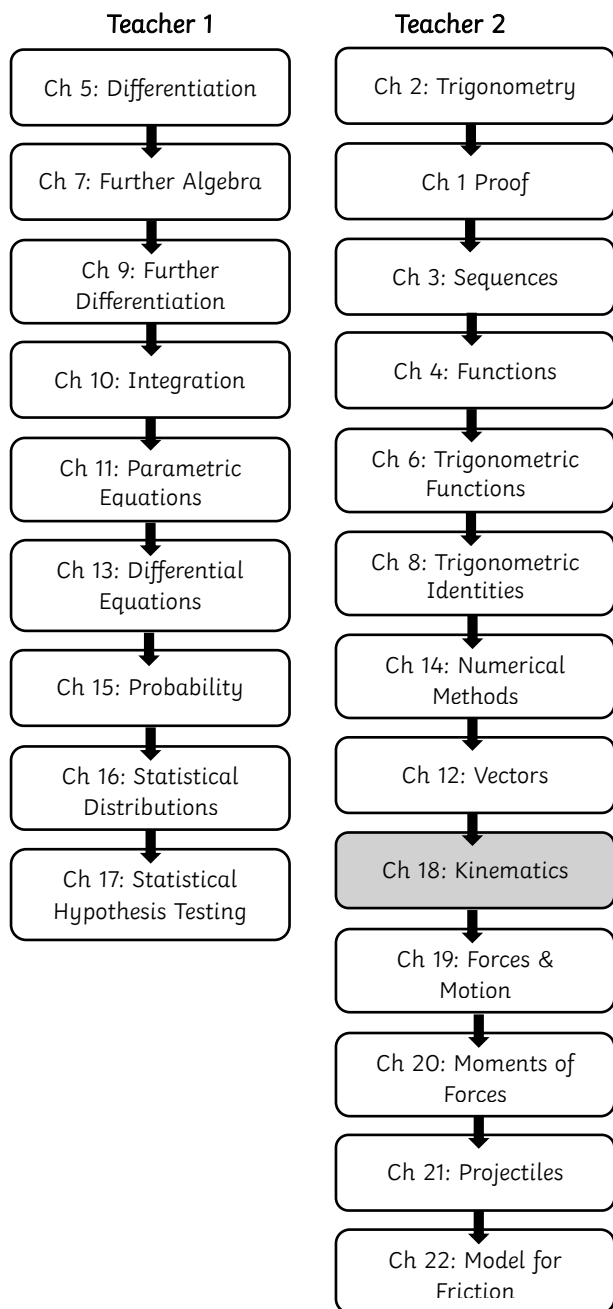
p	10	5	2	1
z	1.645	1.960	2.326	2.576



Specification: OCR Mathematics B (MEI) H640

STATISTICS: STATISTICAL HYPOTHESIS TESTING (2)					
Hypothesis testing for a mean using Normal distribution	MH7	Know that random samples of size n from $X \sim N(\mu, \sigma^2)$ have the sample mean Normally distributed with mean μ and variance $\frac{\sigma^2}{n}$.		Sample mean, \bar{X} Particular value of sample mean, \bar{x} Population mean, μ	Central Limit Theorem
	H8	Be able to carry out a hypothesis test for a single mean using the Normal distribution and be able to interpret the results in context.	In situations where either (a) the population variance is known or (b) the population variance is unknown but the sample size is large Learners may be asked to use a p -value or a critical region. H_0 of form $\mu =$ a particular value, where μ is the population mean. Significance level will be given.		
	H9	Be able to identify the critical and acceptance regions.			

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
STATISTICS: STATISTICAL HYPOTHESIS TESTING (2)					
Informal hypothesis testing for correlation/ association	MH10	Understand correlation as a measure of how close data points lie to a straight line. Understand that a rank correlation coefficient measures the correlation between the data ranks rather than actual data values.	Learners are not required to know the names of particular correlation coefficients.	r	Calculation of correlation coefficient
	H11	Be able to use a given correlation coefficient for a sample to make an inference about correlation or association in the population for given p -value or critical value.	Association refers to a more general relationship between the variables. The (often implicit) null hypothesis is of the form either that there is no correlation or no association in the population. Questions will use an appropriate correlation coefficient and indicate whether correlation or association is being tested for.	Questions may require understanding of notation from software; sufficient guidance will be given in the question.	Knowledge of bivariate Normal distribution
Calculating correlation					
Learners are expected to use technology to work with real data, including the pre-release data. Calculators, spreadsheets and other software will calculate correlation coefficients. Learners may be asked to interpret such correlation coefficients in the examination. The following points should be noted:					
<ul style="list-style-type: none"> A correlation coefficient measures the strength of a linear relationship. A correlation between the ranks of the data values may be used for a more general relationship. Correlation coefficients will only be used for data where both variables are random (not, for example, for time series data where one variable occurs at set intervals). Outliers or distinct sections of data in the scatter diagram can affect the value of the correlation coefficient. 					
Conclusion from a hypothesis test					
Learners are expected to make non-assertive conclusions in context. E.g. "There is not enough evidence to conclude that the proportion of... has increased." E.g. "There is enough evidence to indicate that the probability of has changed." E.g. "There is insufficient evidence to indicate that the true mean of is lower than....." E.g. "There is sufficient evidence to suggest that there is positive correlation between..... and" E.g. "There is not sufficient evidence to suggest that there is association between ... and"					



Chapter 18: Kinematics	
Understand the language of kinematics appropriate to motion in 2 dimensions	
Know the difference between <ul style="list-style-type: none"> displacement, distance from and distance travelled velocity and speed acceleration and magnitude of acceleration 	
Use vectors to model motion in two dimensions <ul style="list-style-type: none"> using calculus using the constant acceleration formulae 	
Find the cartesian equation of the path of a particle when the components of its position vector are given in terms of time <ul style="list-style-type: none"> express t in terms of either x or y substitute for t in the expression for the other coordinate 	
Use vectors to solve problems in kinematics <ul style="list-style-type: none"> Including relative position of one particle from another Including knowing that the velocity vector gives the direction of motion and the acceleration vector gives the direction of resultant force 	
Know that if motion does not start at the origin, s is replaced by $s - s_0$ and r is replaced by $r - r_0$	

Formula Sheet Extract

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two and three dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

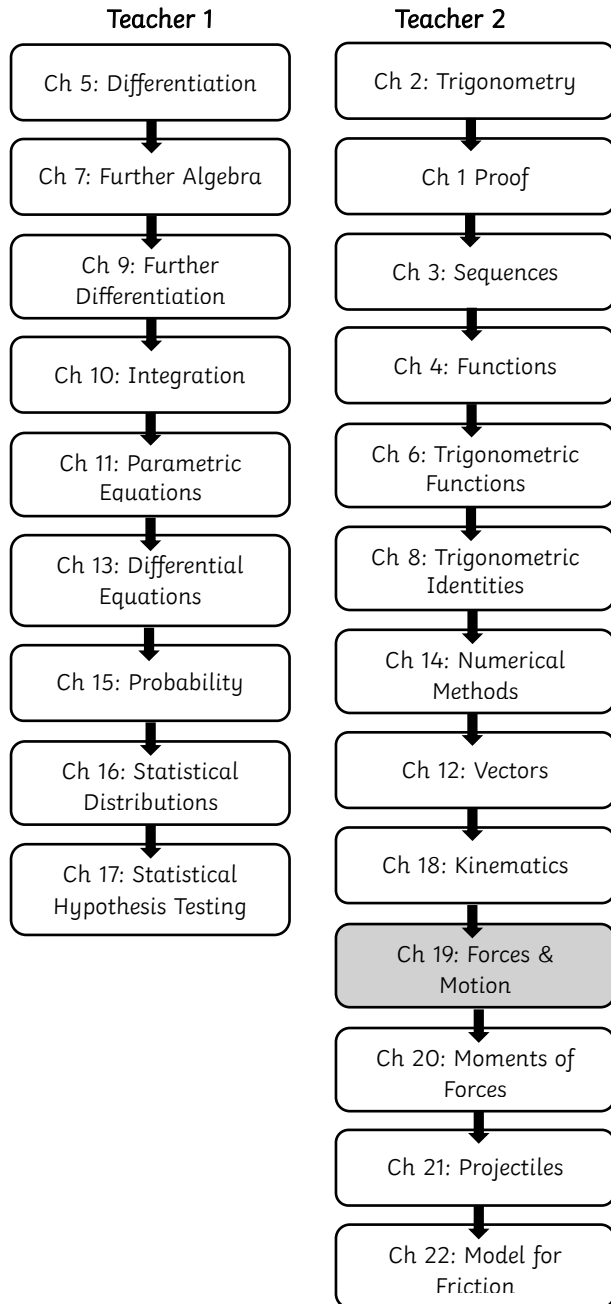
$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

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$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS: KINEMATICS IN 2 DIMENSIONS (2)					
Motion in 2 dimensions	Mk9	Understand the language of kinematics appropriate to motion in 2 dimensions. Know the difference between, displacement, distance from and distance travelled; velocity and speed, and between acceleration and magnitude of acceleration.	Position vector, relative position. Average speed = distance travelled ÷ elapsed time Average velocity = overall displacement ÷ elapsed time		Relative velocity
	k10	Be able to extend the scope of techniques from motion in 1 dimension to that in 2 dimensions by using vectors.	The use of calculus and the use of constant acceleration formulae.	$\mathbf{a} = \dot{\mathbf{v}} = \frac{d\mathbf{v}}{dt}, \mathbf{v} = \dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt}$ $\mathbf{r} = \int \mathbf{v} dt, \mathbf{v} = \int \mathbf{a} dt$ $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$ $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ $\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$	Vector form of $v^2 - u^2 = 2as$
	k11	Be able to find the cartesian equation of the path of a particle when the components of its position vector are given in terms of time.			
	k12	Be able to use vectors to solve problems in kinematics.	Includes relative position of one particle from another. Includes knowing that the velocity vector gives the direction of motion and the acceleration vector gives the direction of resultant force.		

Learning Journey: Year 13 Maths



Personalised Learning Checklist (PLC): Year 13 Maths

Chapter 19: Forces and Motion	
Resolve a force into components	
Find the resultant of several concurrent forces by resolving and adding components <ul style="list-style-type: none"> horizontally and vertically parallel and perpendicular to the inclined plane 	
Know that a particle is in equilibrium if and only if the resultant of the forces acting on it is zero	
Know that a body is in equilibrium under a set of concurrent forces if and only if their resultant is zero	
Know that vectors representing a set of forces in equilibrium sum to zero	
Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium	
Formulate and solve equations for a particle in equilibrium <ul style="list-style-type: none"> by resolving forces in suitable directions by drawing and using a polygon of forces 	
Formulate the equation of motion for a particle moving in a straight line or in a plane, using $F = ma$ where F is the resultant force	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS: FORCES (2)					
Vector treatment of forces	MF6	Be able to resolve a force into components and be able to select suitable directions for resolution. Be able to find the resultant of several concurrent forces by resolving and adding components.	e.g. Horizontally and vertically, or parallel and perpendicular to an inclined plane.		
	F7	Know that a particle is in equilibrium if and only if the resultant of the forces acting on it is zero. Know that a body is in equilibrium under a set of concurrent forces if and only if their resultant is zero.			
	F8	Know that vectors representing a set of forces in equilibrium sum to zero. Know that a closed figure may be drawn to represent the addition of the forces on an object in equilibrium.			
	F9	Be able to formulate and solve equations for a particle in equilibrium: by resolving forces in suitable directions; by drawing and using a polygon of forces.	For example, a triangle of forces.		Non-coplanar forces

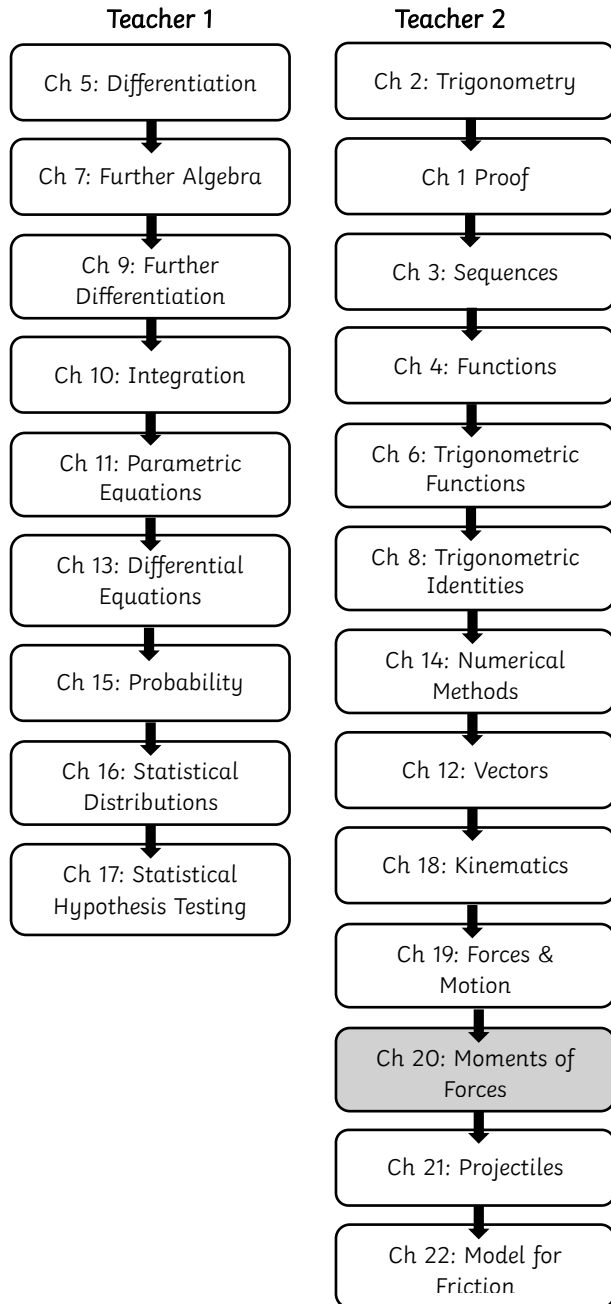
MECHANICS: NEWTON'S LAWS OF MOTION (2)

Newton's laws for a particle	n7	Be able to formulate the equation of motion for a particle moving in a straight line or in a plane.	Including motion under gravity.	$F = ma$ where F is the resultant force. $\mathbf{F} = m\mathbf{a}$ where \mathbf{F} is the resultant force.	Variable mass.
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Newton's laws of motion

- I An object continues in a state of rest or uniform motion in a straight line unless it is acted on by a resultant force.
- II A resultant force \mathbf{F} acting on an object of fixed mass m gives the object an acceleration \mathbf{a} given by $\mathbf{F} = m\mathbf{a}$.
- III When one object exerts a force on another, there is always a reaction which is equal in magnitude and opposite in direction to the acting force.

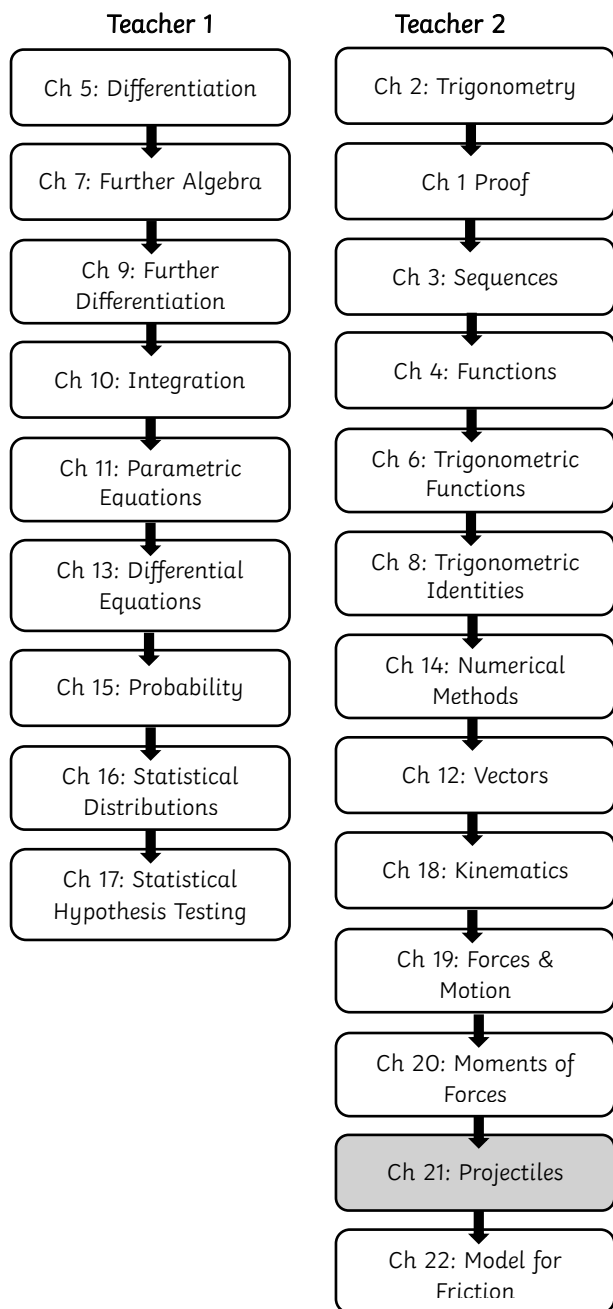
Learning Journey: Year 13 Maths



Personalised Learning Checklist (PLC): Year 13 Maths

Chapter 20: Moments of Forces	
Understand that a system of forces can have a turning effect on a rigid body	
Calculate the moment of a force about a point or axis using the product Fd where d is the perpendicular distance from the O to the line of action of the force	
Know that, for the purpose of calculating its moment, the weight of a body can be taken as acting through a point, which is the centre of mass.	
Know that unit for moment is the newton metre Nm	
Know that anticlockwise moments are usually called positive, and clockwise moments are usually negative	
Know the meaning of the term couple – a pair of forces with zero resultant, but non-zero total moment, therefore causing rotation	
Know that a rigid body is in equilibrium when the resultant force is zero and the sum of the moments about any one point is zero	
Solve problems involving equilibrium of a rigid body	

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS: RIGID BODIES (2)					
<p>This section is an introduction to moments in static contexts. The only situations considered are bodies that may be modelled as (possibly non-uniform) rods and rectangular laminas. The only forces considered are coplanar, and act perpendicular to the rod or to an edge of the lamina. The learning outcome should be read in the light of this restriction.</p> <p>In more advanced work, moments are described as acting about an axis, and learners should be aware of this. Given the restrictions on the situations considered, however, moments may be described as acting about a point, with an implied axis perpendicular to the plane in which the forces are acting. This is consistent with the approach used to describe rotations in 2-D.</p>					
Rigid bodies in equilibrium	MF13	Be able to calculate the moment of a force about a point or axis.		Units of moment are N m.	Vector treatment.
	F14	Understand that a rigid body is in equilibrium when the resultant force is zero and the sum of the moments about any one point is zero.			
	F15	Understand that a system of forces can have a turning effect on a rigid body.		Moment	
	F16	Know that, for the purpose of calculating its moment, the weight of a body can be taken as acting through a point.	<p>The point is the centre of mass of the body.</p> <p>Questions will be restricted to cases where the centre of mass is given or can be found using symmetry or can be found from consideration of moments.</p>	Uniform	Finding the centre of mass of a composite body.



Chapter 21: Projectiles	
Model motion under gravity in a vertical plane using vectors	
Formulate the equations of motion of a projectile using vectors	
Know that the standard modelling assumptions for projectile motion are as follows <ul style="list-style-type: none"> • Air resistance has no effect on its motion • The projectile is a particle • It is not powered • Gravity is always in the same direction and constant 	
Find the position and velocity at any time of a projectile	
Find range of a projectile using $y = 0$ and find maximum height using $v_y = 0$	
Find the initial velocity of a projectile given sufficient information	
Formulate the equations of motion of a projectile using vectors	
Eliminate time from the component equations that give the horizontal and vertical displacement in terms of time to obtain the equation of the trajectory	
Solve simple problems involving projectiles	
Find equations for the motion of a projectile	

Formulas to Learn

When initial position is $(0, 0)$ and the angle of projection is α

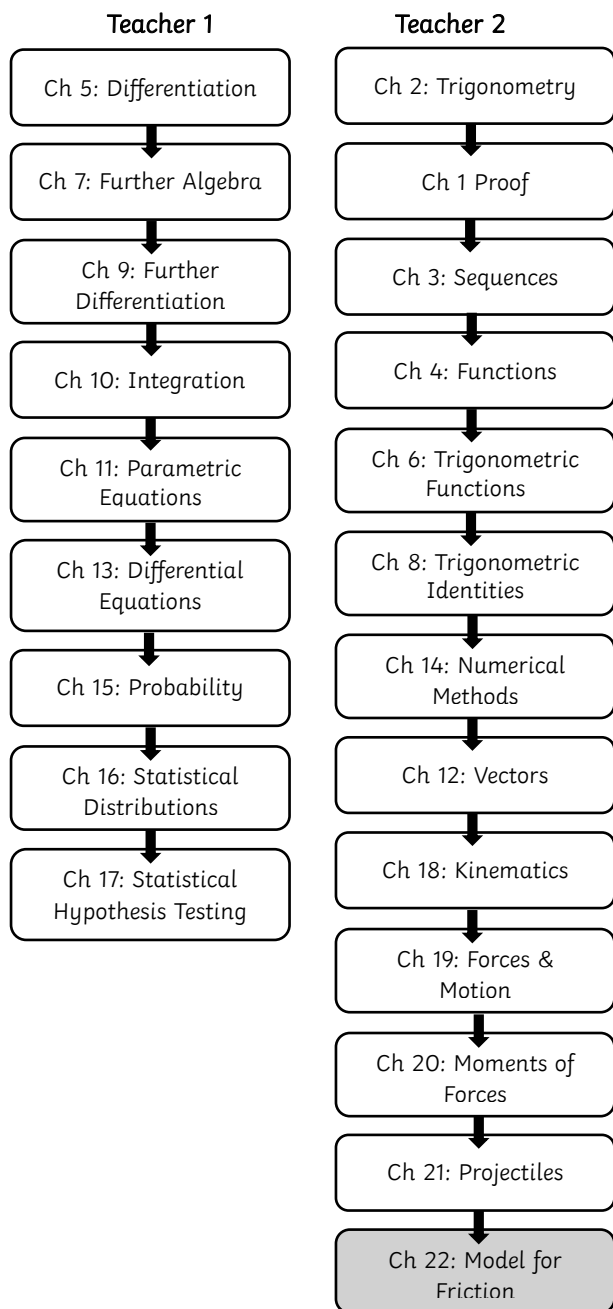
Initial velocity, $\mathbf{u} = \begin{pmatrix} u \cos \alpha \\ u \sin \alpha \end{pmatrix}$

Acceleration, $\mathbf{a} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$

At time, t , velocity $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ and $v_x = u \cos \alpha$, $v_y = u \sin \alpha - gt$

Displacement, $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ and $x = ut \cos \alpha$, $y = ut \sin \alpha - \frac{1}{2}gt^2$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS: PROJECTILES (2)					
Motion under gravity in 2 dimensions	My1	Be able to model motion under gravity in a vertical plane using vectors. Be able to formulate the equations of motion of a projectile using vectors.	Standard modelling assumptions for projectile motion are as follows. <ul style="list-style-type: none"> • No air resistance. • The projectile is a particle. • Horizontal distance travelled is small enough to assume that gravity is always in the same direction. • Vertical distance travelled is small enough to assume that gravity is constant. 		Calculations involving air resistance
	y2	Know how to find the position and velocity at any time of a projectile and find range and maximum height.			
	y3	Be able to find the initial velocity of a projectile given sufficient information.			
	y4	Be able to eliminate time from the component equations that give the horizontal and vertical displacement in terms of time to obtain the equation of the trajectory.			
	y5	Be able to solve simple problems involving projectiles.			Maximum range on inclined plane Bounding parabola



Chapter 22: A Model for Friction	
Understand that the overall contact force between surfaces may be expressed in terms of a frictional force F and a normal contact force R , and be able to draw an appropriate force diagram	
Understand that the normal contact force cannot be negative	
Know that friction acts in the direction to oppose sliding	
Understand the following modelling assumptions <ul style="list-style-type: none"> • Smooth is used to mean that friction may be ignored • Rough indicates that friction must be taken into account 	
Model the frictional force as $F \leq \mu R$ where μ is the coefficient of friction $F < \mu R$ when there is no sliding (except in limiting equilibrium) $F = \mu R$ in limiting equilibrium (at the point of sliding) $F = \mu R$ when sliding occurs	
Be able to apply Newton's Laws to problems involving friction	
Know the size of the normal reaction, and possibly friction, is affected by any other force with a component perpendicular to the surface on which sliding may occur	

Formulas to Learn

Mechanics

Forces and Equilibrium

Weight = mass \times g

Friction: $F \leq \mu R$

Newton's second law in the form: $F = ma$

Specification	Ref.	Learning outcomes	Notes	Notation	Exclusions
MECHANICS: FORCES (2)					
Frictional force and normal contact force	F10	Understand that the overall contact force between surfaces may be expressed in terms of a frictional force and a normal contact force and be able to draw an appropriate force diagram. Understand that the normal contact force cannot be negative.	Understand the following modelling assumptions. <ul style="list-style-type: none"> • Smooth is used to mean that friction may be ignored. • Rough indicates that friction must be taken into account. 	Normal reaction.	
	F11	Understand that the frictional force may be modelled by $F \leq \mu R$ and that friction acts in the direction to oppose sliding. Model friction using $F = \mu R$ when sliding occurs.		Coefficient of friction = μ Limiting friction, static equilibrium	The term angle of friction.
	F12	Be able to apply Newton's Laws to problems involving friction.			